Introduction to Mechanisms and Kinematics

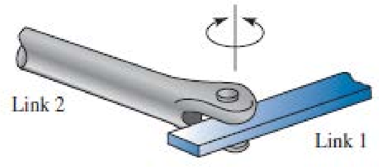
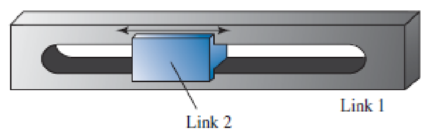
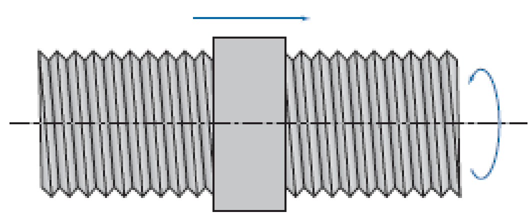
Basic Definitions

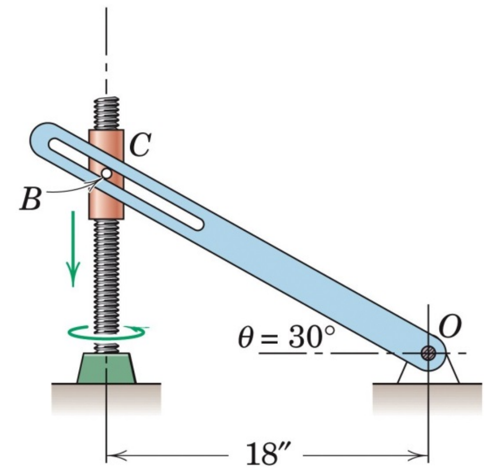
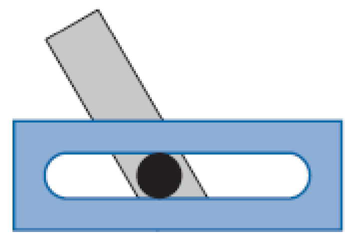
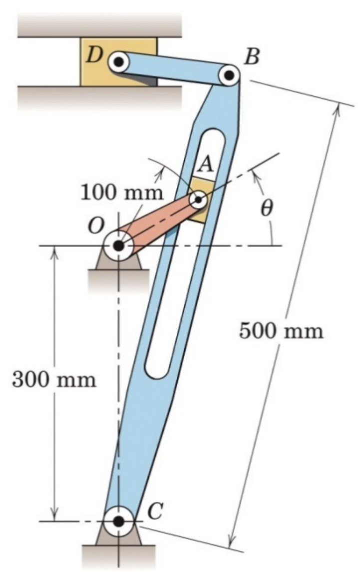
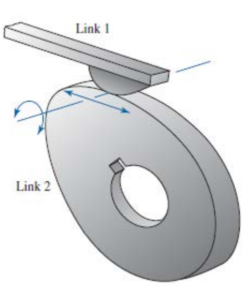
- Machines are devices used to accomplish work. A mechanism is the heart of a machine. It is the mechanical portion of a machine that has the function of transferring motion and forces from a power source to an output.
- Mechanisms are assemblage of rigid members (links) connected together by joints (also referred to as Mechanical linkage or linkage).
  - Links are the individual parts of the mechanism. They are considered rigid bodies and are connected with other links to transmit motion and force. Elastic parts, such as springs, are not rigid and, therefore, are not considered links. They have no effect on the kinematics of a mechanism and are usually ignored during kinematic analysis. They do supply forces and must be included during the dynamic force portion of analysis

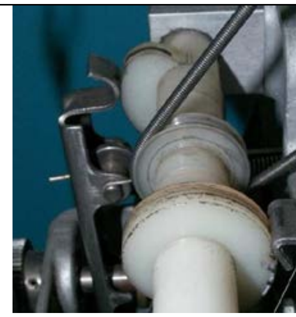
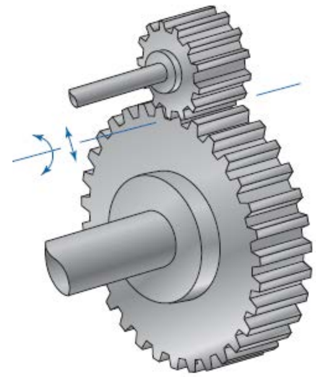
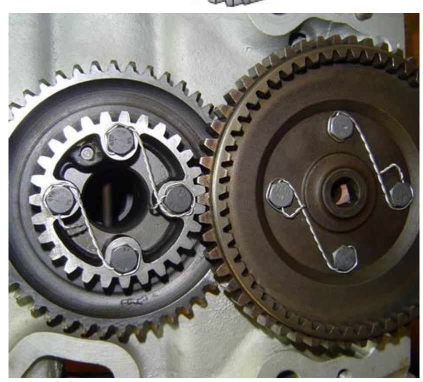
Example

The adjustable height platform (on the right) is driven by hydraulic cylinders. Although the entire device could be called a machine, the parts that take the power from the cylinders and drive the raising and lowering of the platform are mechanisms.

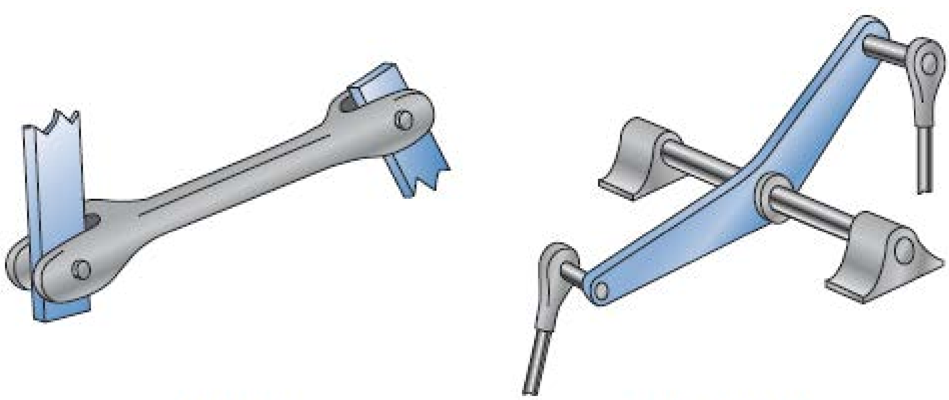
  - A joint is a movable connection between links and allows relative motion between the links. Some common joints

Joint	Description	
Revolute, pin or hinge	Allows pure rotation between the two links that it connects.	 
Sliding, piston or prismatic	Allows linear sliding between the links that it connects	 
Screw joint	Permits two relative but dependent motions between the links being joined.	 

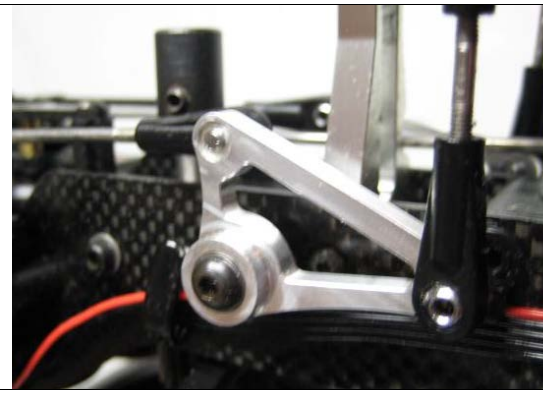
		
Pin in a slot Joint	Permits the two links to slide and rotate relative to one another	 
Cam	Allows for both rotation and sliding between the two links that it connects. Because of the complex motion permitted, the cam connection is called a higher-order joint, also called half join	

		
Gear	A gear connection also allows rotation and sliding between two gears as their teeth mesh.	 

- A simple link is a rigid body that contains only two joints, which connect it to other links. A complex link is a rigid body that contains more than two joints.

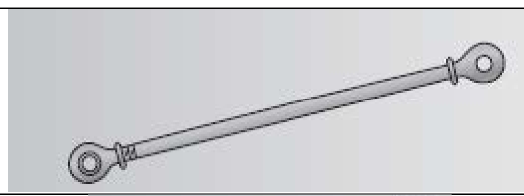
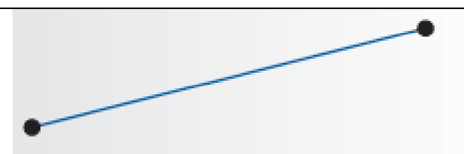
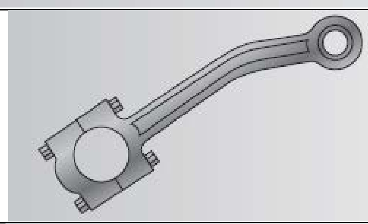
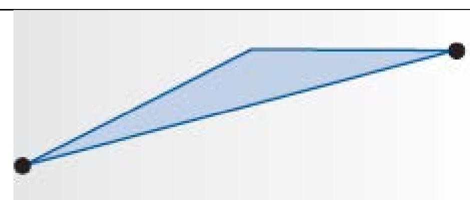
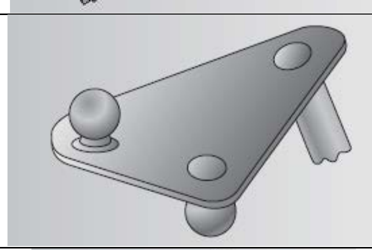
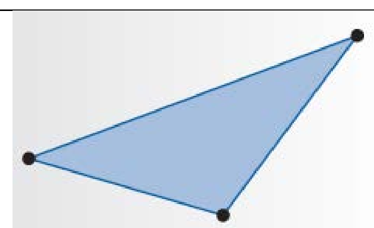
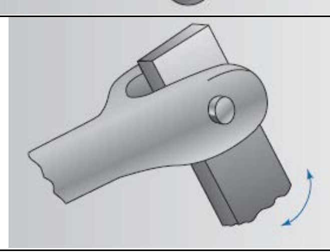
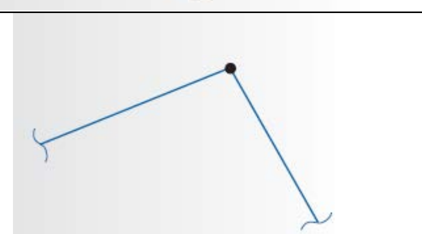


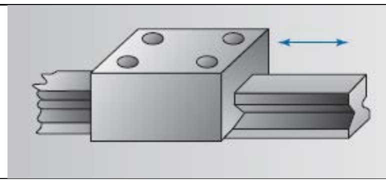
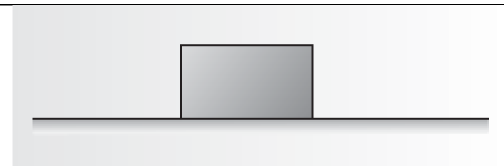
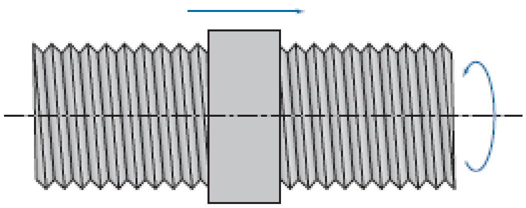
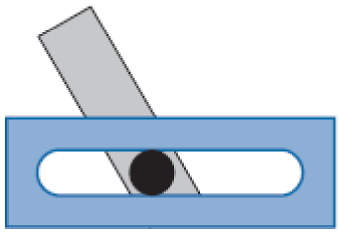
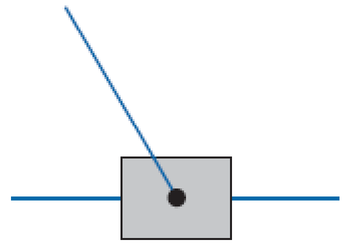
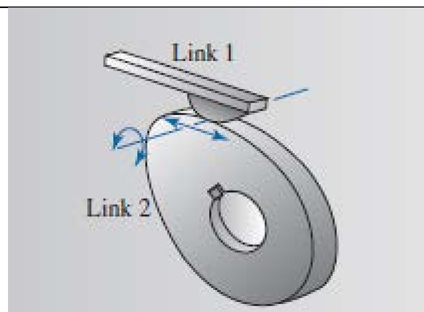
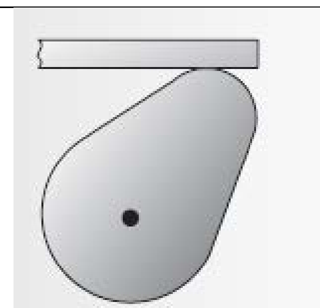
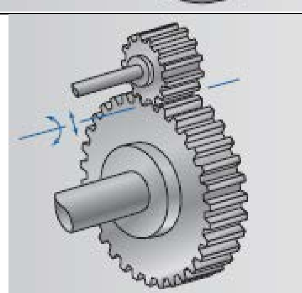
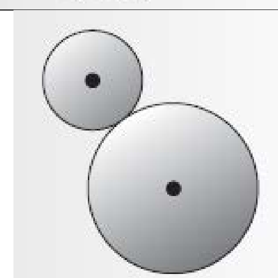
(a) Simple link (b) Complex link

Kinematic Diagram

Kinematic analysis involves determination of position, displacement, rotation, speed, velocity, and acceleration of a mechanism. In analyzing the motion of a mechanism, it is often convenient to represent the parts in skeleton form (also referred to as kinematic diagram) so that only the dimensions that affect the motion are shown. A standard representation is used for the components of a mechanism as listed in the table.

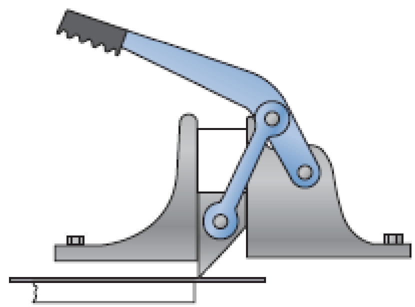
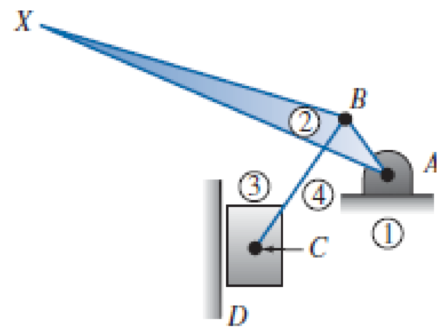
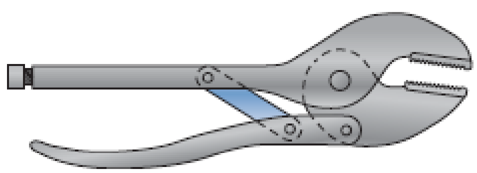
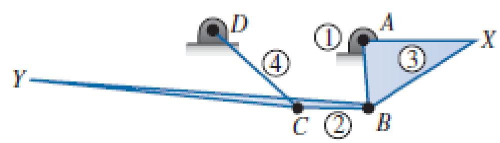
Component	Typical form	Kinematic representation
Simple link		
Simple link with point of interest		
Complex link		
Pin joint		

Slider joint		
Screw joint		
Pin in a slot joint		
Cam joint		
Gear Joint		

- A kinematic diagram should be drawn to a scale proportional to the actual mechanism. For convenient reference, the links are numbered, starting with the frame, which serves as the frame of reference for the motion of all other parts. To avoid confusion, the joints should be lettered.

Examples

- In the first diagram, the motion of the bar handle is a point of interest
- In the second diagram, the motion of the end of the bottom jaw and the lower handle are points of interest.

Typical form	Kinematic representation
	
	

Mobility

The degree of freedom of a linkage is the number of independent inputs required to precisely position all links of the mechanism with respect to the frame. It can also be defined as the number of actuators needed to operate the mechanism. The number of degrees of freedom of a mechanism is also called the mobility, and it is given the symbol (M).

- When the configuration of a mechanism is completely defined by positioning one link, that system has one degree of freedom.

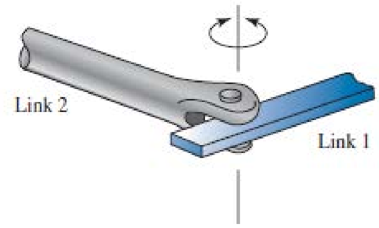
Mobility in the planar case

A body moving freely in planar motion has three degrees of freedom (two translations on the plane and one rotation about an axis perpendicular to the plane). Suppose that in a given linkage there are (n) links.

- If they're all free to move independently, the system has Mobility (M = 3n).
- If one link is chosen as the frame link, it is fixed to the base reference frame and loses all its degrees of freedom. Therefore, the total Mobility of the system is (M = 3(n-1)).
- If a joint with connectivity $\mathbf{f_i}$ (f_i degrees of freedom of the joint) is formed between two bodies the mobility of the system is diminished since those two bodies originally had three degrees of freedom of motion relative to one another. After formation of the joint they have only f_i degrees of freedom of relative motion. Hence the reduction in the system mobility is $\mathbf{3 - f_i}$.

Example

Without the pin joint, each link has 3 degrees of freedom. The pin joint however only allows free relative rotation (one degree of freedom) between the links. Relative translation is omitted (two free translations were omitted)



- If joints continue to be formed until there are **j** joints, the loss of system mobility is

$$\sum_{i=1}^{j}(3-f_i)$$

The total mobility of the linkage then becomes

$$M = 3(n-1) - \sum_{i=1}^{j}(3-f_i)$$

The above equation can be put in a simpler form also known as the **Gruebler** equation

$$M = 3(n-1) - 2j_p - j_h$$

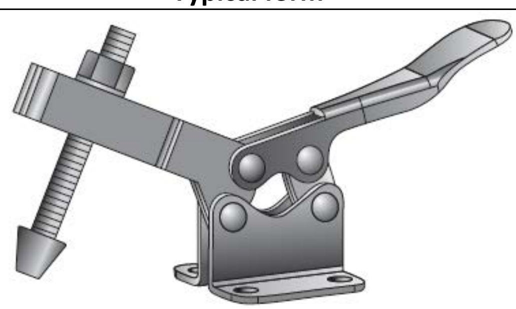
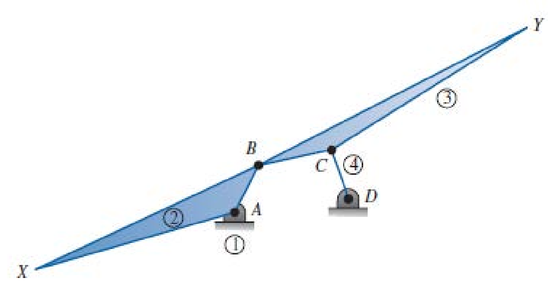
Where

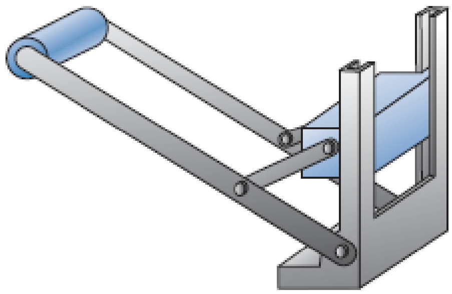
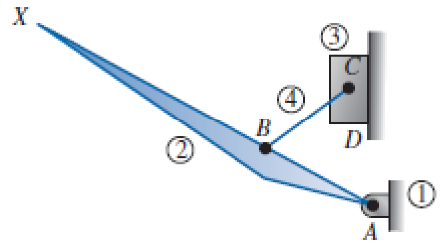
n is the total number of links in the mechanism

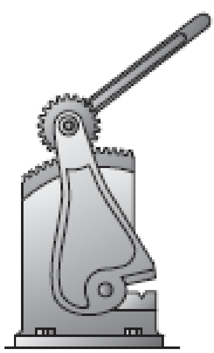
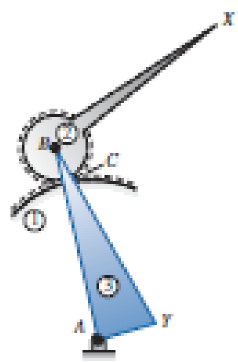
j_p is the number of primary joints (pins or sliding joints)

j_h is the number of higher order joints (cam or gear joints)

Examples

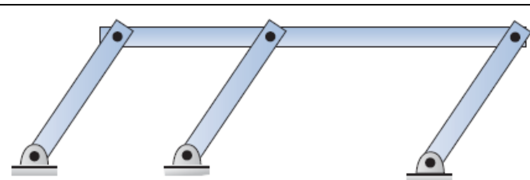
Typical form	Kinematic representation
	
n = 4 (four members), one of the members is frame, 4 pin joints with one degree of freedom $M = 3(4-1)-(3-1)-(3-1)-(3-1)-(3-1) = 3(4-1)-4(2) = 1$	

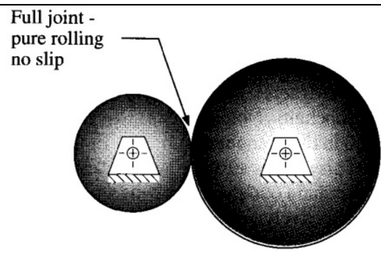
<table>
<tr><th>Typical form</th><th>Kinematic representation</th></tr>
<tr><td></td><td>
</td></tr>
<tr><td colspan="2">n = 4 (four members), one of the members is frame, 3 pin joints with one degree of freedom and one slider joint with one degree of freedom
$$M = 3(4-1) - (3-1) - (3-1) - (3-1) - (3-1) = 3(4-1) - 4(2) = 1$$</td></tr>
</table>

<table>
<tr><th>Typical form</th><th>Kinematic representation</th></tr>
<tr><td></td><td>
</td></tr>
<tr><td colspan="2">n = 3 (three members), one of the members is frame, 2 pin joints with one degree of freedom and one higher order joint
$$M = 3(3-1) - (3-1) - (3-1) - (3-2) = 1$$</td></tr>
</table>

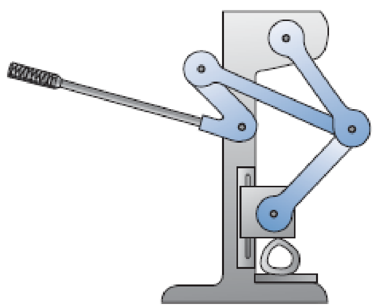
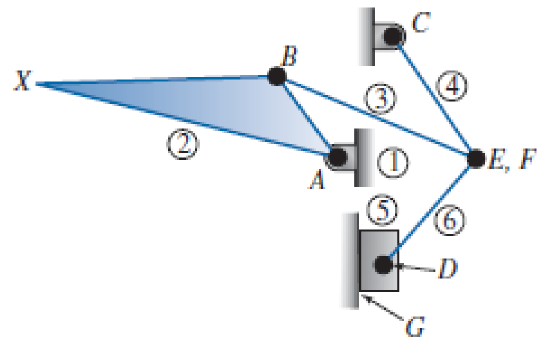
Notes:

- Because the mobility equation pays no attention to link size or shape, it can give misleading results in the face of unique geometric configurations. For instance

<table>
<tr><td></td><td>Using the mobility equation we find
$$M = 3(5-1) - 6(3-1) = 0$$
Because the nodes are equi-spaced and the three links are of equal length, the mobility of the mechanism is equal to 1 rather than 0</td></tr>
</table>

	Assuming enough friction between the wheels to allow for rolling without slipping, the joint between the wheels has one degree of freedom. Using the mobility equation we get $M = 3(3-1) - 3(3-1) = 0$ The linkage however has one degree of freedom

➢ When three links come together at a common pin, the joint must be modeled as two pins

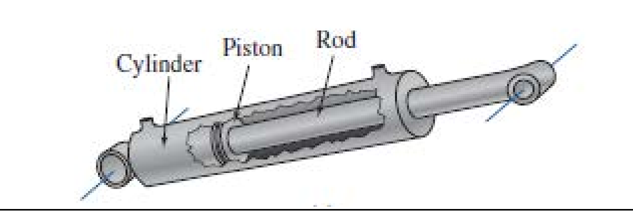
	
$Mobility = 3(6-1) - 6(3-1) - (3-1) = 1$	

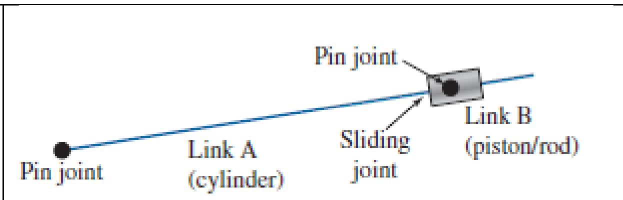
Actuators and Drivers

In order to operate a mechanism, an actuator, or driver device, is required to provide the input motion and energy. To precisely operate a mechanism, one driver is required for each degree of freedom exhibited. Many different actuators are used in industrial and commercial machines and mechanisms. Some of the more common ones

- **Electric motors (AC)**
  - Provide continuous rotary motion
  - Can only rotate at specific speed dependent on the frequency of the power line.
  - Single phase motors are available from 1/50 to 2 hp. Three phase motors are available from 1/4 to 500 hp
- **Electric motors (DC)**
  - Provide continuous rotary motion
  - Require power from a generator or a battery
  - Allow control of speed and direction.
  - Can achieve speeds up to 30,000 rpm
- **Engines**
  - Provide continuous rotary motion
  - Rely on combustion of fuel, the speed can be anywhere from 1000 to 8000 rpm
- **Servomotors**
  - Coupled with a controller to produce a programmed motion.
  - The controller uses sensors to provide feedback information on the position, velocity, and acceleration.
  - Characterized by lower power capacity than non-servomotors.

- **Air or Hydraulic motors**
  - Produce continuous rotary motion
  - Require compressed air or a hydraulic source
- **Hydraulic or pneumatic cylinders**
  - Produce limited linear stroke
  - Require air or hydraulic sources
  - The cylinder unit contains a rod and piston assembly that slides relative to the cylinder
  - The common kinematic representation for a hydraulic or pneumatic cylinder is

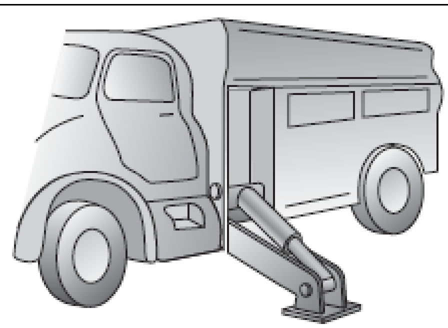


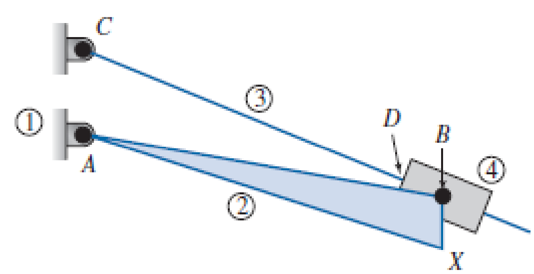


- **Screw actuators**
  - Produce limited linear stroke
  - Consist of a motor rotating a screw. A mating nut provides the linear motion
  - The kinematic diagram of screw actuators is similar to the cylinders
- **Manual or hand operated actuator Actuators**

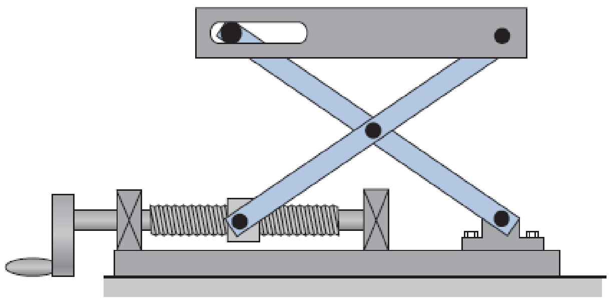
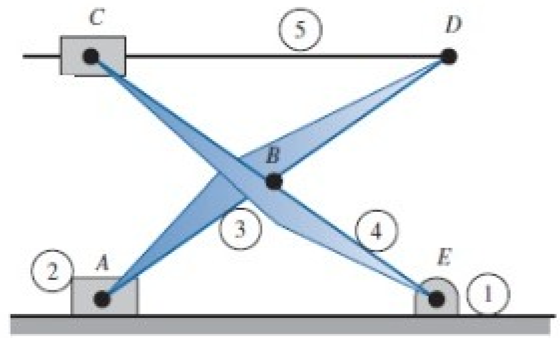
Examples:

➢ For the outrigger in the figure below, the hydraulic cylinder is pinned to the foot to stabilize the truck.





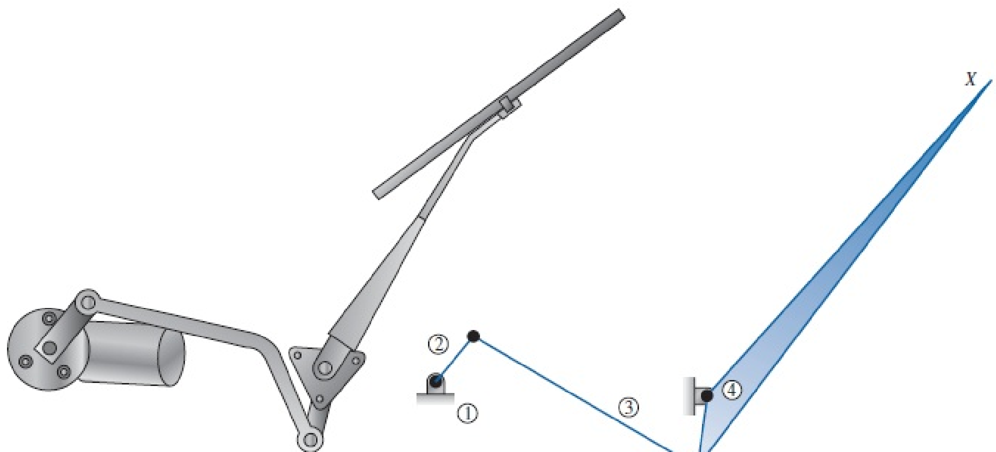
$$\text{Mobility} = 3\,(4-1) - (3-1) - (3-1) - (3-1) - (3-1) = 1$$

<table>
<tr><td></td><td>
</td></tr>
<tr><td colspan="2">$Mobility = 3(5-1) - 4(3-1) - (3-2) - (3-1) = 1$</td></tr>
</table>

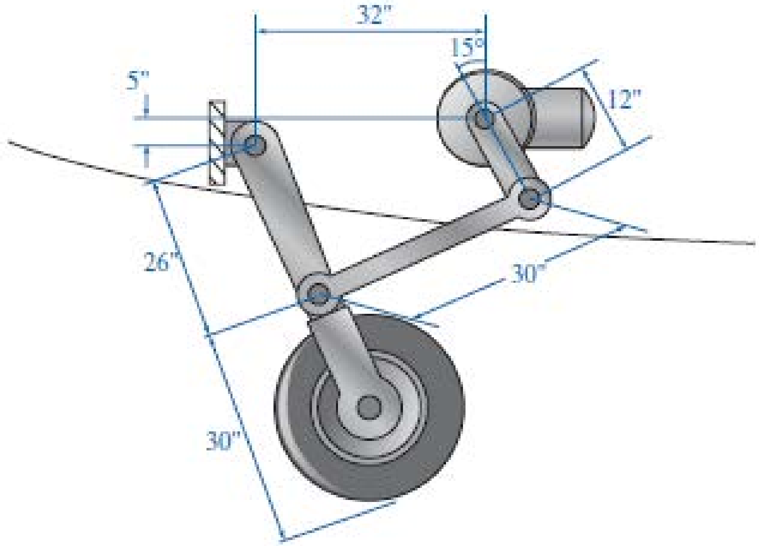
The four-bar mechanism

A four bar linkage is a mechanism made of four links, one being designed as the frame and connected by four pins.

Example1: Four bar window wiper mechanism



Example2: Nose wheel mechanism



The link that is unable to move is referred to as the frame. Typically, the pivoted link that is connected to the driver or power source is called the input link. The other pivoted link that is attached to the frame is designated the output link or follower. The coupler or connecting arm "couples" the motion of the input link to the output link.

The mobility of these mechanisms is

$$M = 3(4-1) - 4(3-1) = 1$$

Because the four-bar mechanism has one degree of freedom, it is constrained or fully operated with one driver. The wiper and Nose wheel systems above are activated by a single motor.

<u>Grashof's Criterion</u>

Depending on the sizes of the links in the four-bar mechanism, the mechanism may act as one of the following

- Double crank where If one of the pivoted links is rotated continuously, the other pivoted link will also rotate continuously.
- Crank rocker where the output link will oscillate between limits.
- Double rocker where both input and output links are constrained to oscillate between limits.
- Change point where can be all the links can be positioned to becoming collinear.
- Triple rocker where all three moving links rock

A criterion known as Grashof's theorem can help differentiate among the different configurations

If

S is the length of the shortest link
L the length of the longest link
P and Q the lengths of the other two links

Then

- Grashof's theorem states that a four-bar mechanism has at least one revolving link if:
$$S + L \leq P + Q$$

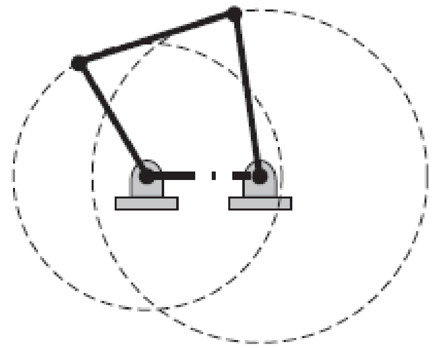
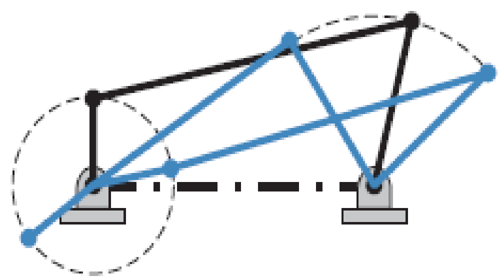
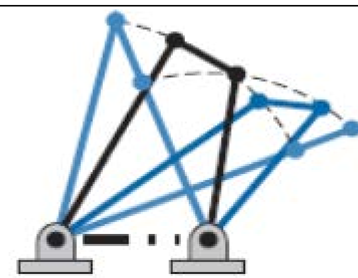
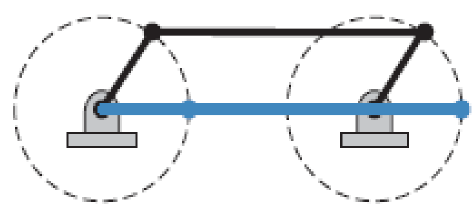
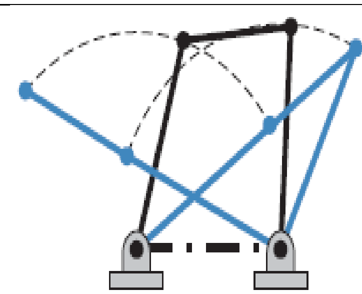
- Conversely, the three non-fixed links will merely rock if:

$$S + L > P + Q$$

The results of Grashof's theorem are summarized the table below

TABLE 1.2 Categories of Four-Bar Mechanisms

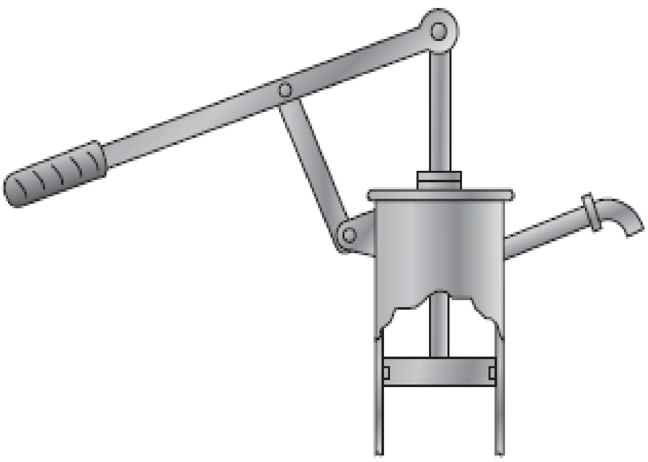
Case	Criteria	Shortest Link	Category
1	$s + l < p + q$	Frame	Double crank
2	$s + l < p + q$	Side	Crank-rocker
3	$s + l < p + q$	Coupler	Double rocker
4	$s + l = p + q$	Any	Change point
5	$s + l > p + q$	Any	Triple rocker

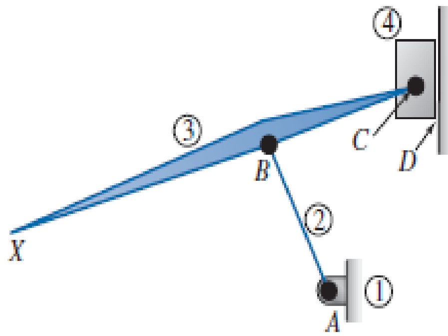
The double crank mechanism As specified in the criteria of Case 1 of the table, it has the shortest link of the four-bar mechanism configured as the frame. If one of the pivoted links is rotated continuously, the other pivoted link will also rotate continuously. Thus, the two pivoted links are both able to rotate through a full revolution. The double crank mechanism is also called a drag link mechanism.	
Crank-Rocker As specified in the criteria of Case 2 of the table, it has the shortest link of the four-bar mechanism configured adjacent to the frame. If this shortest link is continuously rotated, the output link will oscillate between limits. Thus, the shortest link is called the crank, and the output link is called the rocker. The wiper system previously mentioned is designed to be a crank-rocker. As the motor continuously rotates the input link, the output link oscillates, or “rocks.” The wiper arm and blade are firmly attached to the output link, oscillating the wiper across a windshield.	
Double Rocker As specified in the criteria of Case 3 of the table, it has the link opposite the shortest link of the four-bar mechanism configured as the frame. In this configuration, neither link connected to the frame will be able to complete a full revolution. Thus, both input and output links are constrained to oscillate between limits, and are called rockers.	
Change Point Mechanism As specified in the criteria of Case 4 of the table, the sum of two sides is the same as the sum of the other two. Having this equality, the change point mechanism can be positioned such that all the links become collinear.	
Triple Rocker The triple rocker has no links that are able to complete a full revolution. Thus, all three moving links rock.	

Other mechanisms

The Slider Crank Mechanism

This mechanism consists of a combination of four links, with one being designated as the frame. The links are connected by three pin joints and one sliding joint. A mechanism that drives a manual water pump is an example. The corresponding kinematic diagram is also given.





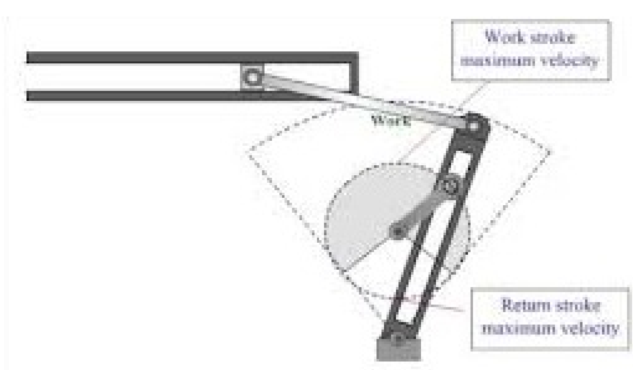
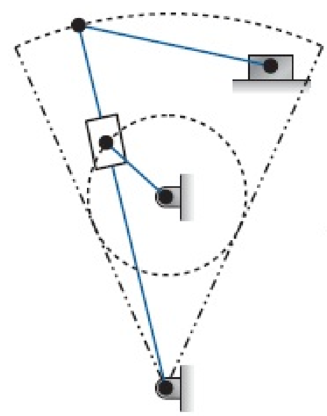
The mobility of this mechanism is

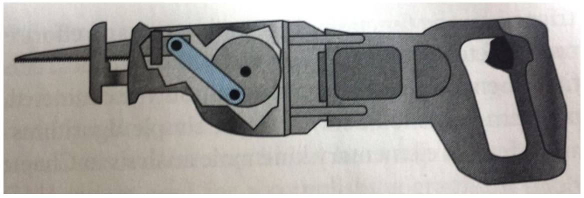
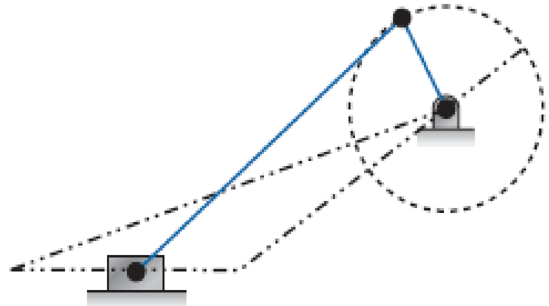
$$M = 3(4-1) - 2*3 - 2*1 = 1$$

Because the slider-crank mechanism has one degree of freedom, it is constrained or fully operated with one driver. The pump is activated manually by pushing on the handle.

Quick return mechanism

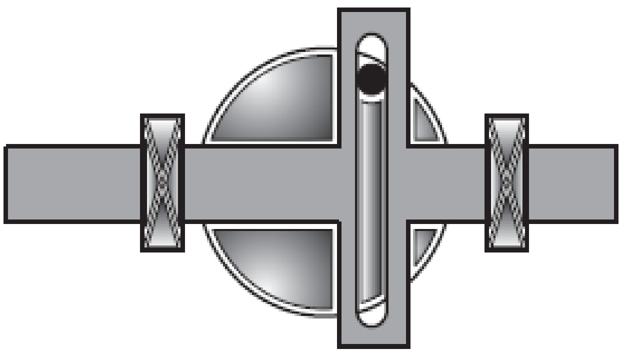
Quick-return mechanisms exhibit a faster stroke in one direction than the other when driven at constant speed with a rotational actuator. They are commonly used on machine tools that require a slow cutting stroke and a fast return stroke.

 crank-shaper linkage	

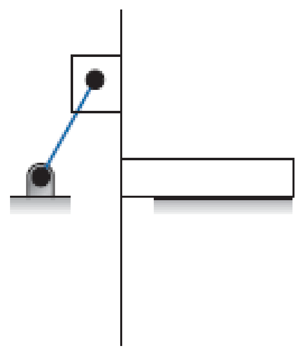
 offset slider-crank in a reciprocating saw	

Scotch Yoke Mechanism

A scotch yoke mechanism converts rotational motion to linear sliding motion, or vice Versa. A pin on a rotating link is engaged in the slot of a sliding yoke. With regards to the input and output motion, the scotch yoke is similar to a slider-crank, but the linear sliding motion is pure sinusoidal. In comparison to the slider-crank, the scotch yoke has the advantage of smaller size and fewer moving parts, but can experience rapid wear in the slot.



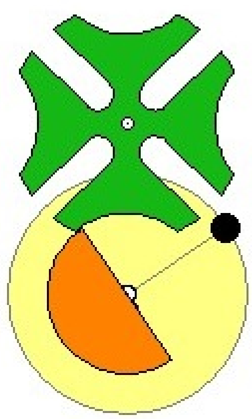
(a) Actual mechanism



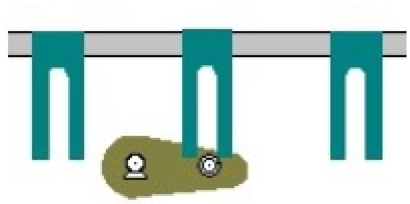
(b) Kinematic diagram

Geneva mechanism

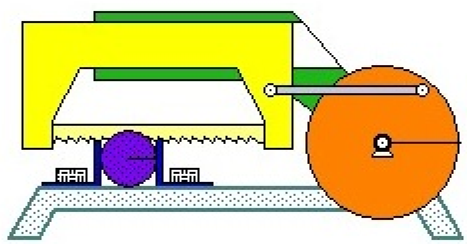
A Geneva mechanism converts the motion of a continuous rotating crank to a intermittent motion of the Geneva wheel. For a four-slot wheel, the Geneva wheel rotates 90 degrees for a complete rotation of a crank.



Linear Geneva
As the crank rotates continuously, the output slider moves intermittently in the linear Geneva mechanism.



Power Hacksaw
This saw (in yellow) has reciprocating motion inside the arm (in green), driven by a rotating disk (in orange) with a coupler (in grey). This is a crank slider mechanism. The arm pivots at the center of the disk to feed the saw downward to cut the metal bar (in purple).



Oil field pump
As the motor turns the crank (in orange), the walking beam (in yellow) oscillates. The pumping (sucker) rod, which is immersed in the oil, is connected to the horse-head of the walking beam by a cable. Therefore, the oscillation of the walking beam is converted to the reciprocating motion of the pumping rod to pump oil.