# HW2 - Ch 3

Q1

a) $G(f) = \dfrac{4 - 2e^{-j\omega} - 2e^{-j2\omega}}{j\omega}$

b) $\mathcal{F}[g(-t)] = G(-\omega) = \dfrac{4 - 2e^{j\omega} - 2e^{j2\omega}}{-j\omega}$

$\mathcal{F}[g(2t)] = \dfrac{1}{|a|} G\left(\dfrac{\omega}{a}\right) = \dfrac{1}{2}\left[\dfrac{4 - 2e^{-j\frac{\omega}{2}} - 2e^{-j\omega}}{j\frac{\omega}{2}}\right]$

c) $g(-t)$: the spectral magnitude is the same, but the phase is inverted.

$g(2t)$: The spectral bandwidth expand by a factor of two.

Q2) a) $g(t) = \displaystyle\int_{-B}^{B} e^{-j2\pi f t_0} e^{j2\pi f t}\, df = 2B \operatorname{sinc}[2\pi B(t - t_0)]$

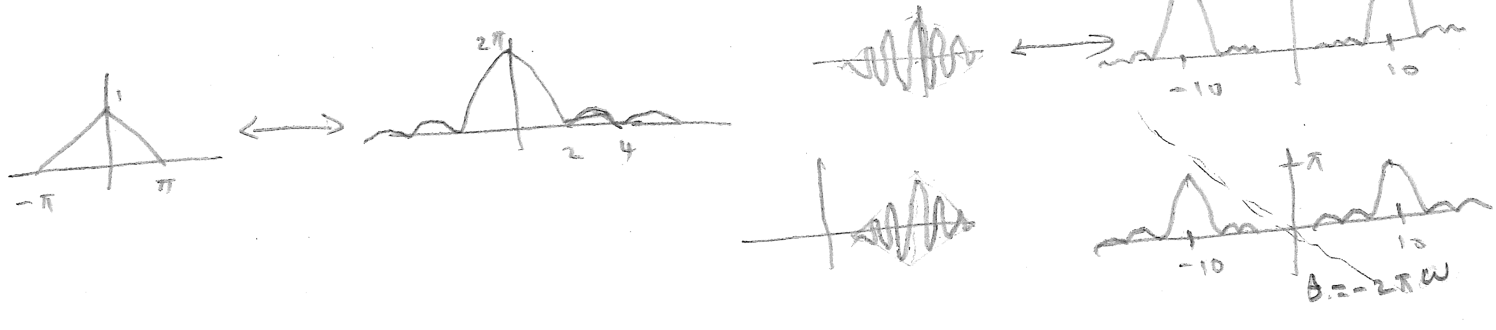
b) $g(t) = \displaystyle\int_{-B}^{0} j e^{j2\pi f t}\, df - \int_{0}^{B} j e^{j2\pi f t}\, df = \dfrac{1 - \cos 2\pi B t}{\pi t}$

Q3) a) $\Delta\left(\dfrac{t}{2\pi}\right) \cos 10t \longleftrightarrow \dfrac{\pi}{2}\left[\operatorname{sinc}^2(\pi^2 f - 5\pi) + \operatorname{sinc}^2(\pi^2 f + 5\pi)\right]$

b) delay the above signal by $2\pi$ result in FT as

$G(f) = \dfrac{\pi}{2}\left[\operatorname{sinc}^2(\pi^2 f - 5\pi) + \operatorname{sinc}^2(\pi^2 f + 5\pi)\right] e^{-j4\pi^2 f}$

c) $G(f) = \dfrac{\pi}{2}\left[\operatorname{sinc}(2\pi^2 f - 10\pi) + \operatorname{sinc}(2\pi^2 f + 10\pi)\right] e^{-j4\pi^2 f}$

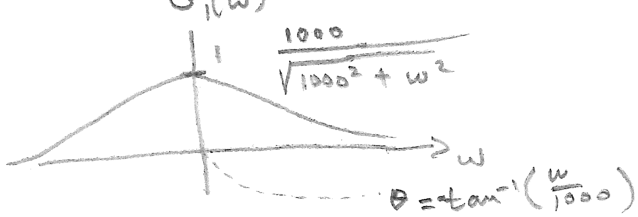


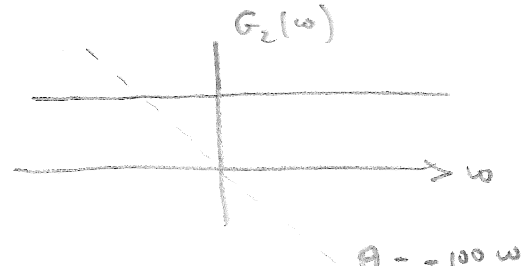
Q4

HW2

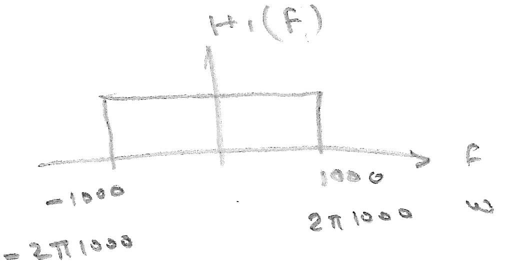
2

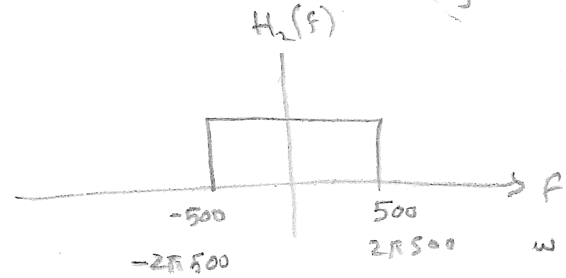
a)



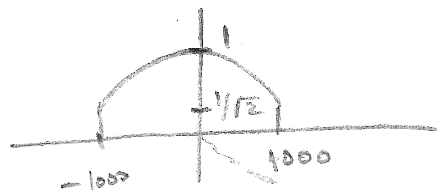



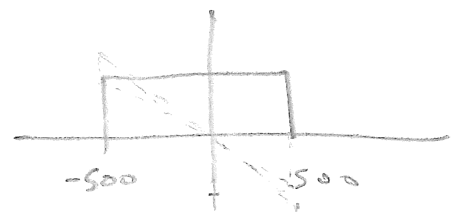

b)




c)




d) $BW_{Y_1} = 1000\ Hz$ $\quad BW_{Y_2} = 500\ Hz$ $\quad BW_Y = 1000 + 500 = 1500\ Hz$

Q5) Review example solved in the class and it is in the textbook.

Hint: percent variation between two values $H_{max}$ and $H_{min}$ can be calculated as

$$\frac{[|H_{max}| - |H_{min}|]}{\frac{1}{2}[|H_{max}| + |H_{min}|]} * 100\%$$

$B < 50\ Hz$

Q6) $H(f)\, H_{eq}(f) = 1$ $\qquad H_{eq}(f) = \frac{1}{1 + \alpha e^{-j2\pi f \Delta t}}$

↑ channel

$a_i = (-1)^i \alpha^i$

Q7) Delay does not affect the energy density function

a) $\Psi_g(f) = 144\pi^2 e^{-4\pi a|f|}$

b) $\int_0^B \Psi_g(f)\,df = 0.99 \int_0^\infty \Psi_g(f)\,df \Rightarrow B = \frac{\ln 100}{4\pi a}$

$B =$

8) $0 < \tau < T_b/2$

$$R_x(\tau) = \lim_{N\to\infty} \frac{1}{NT_b}\left[\frac{N}{2}\left(\frac{T_b}{2} - \tau\right)\right]$$

$$R_x(\tau) = \frac{1}{2}\left(\frac{1}{2} - \frac{|\tau|}{T_b}\right) \qquad |\tau| < \frac{T_b}{2}$$

for $\tau > T_b/2$ & $\tau < T_b$

$$R_x(\tau) = \lim_{N\to\infty} \frac{1}{NT_b}\left[\frac{N}{4}\left(\tau - \frac{T_b}{2}\right)\right] = \frac{1}{4}\left(\frac{\tau}{T_b} - \frac{1}{2}\right), \quad \tau > \frac{T_b}{2}$$

$$S_x(\omega) = \mathcal{F}\left[R_x(\tau)\right] = \frac{T_b}{16}\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right) + \frac{\pi}{8}\sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{\pi n}{2}\right)\delta(\omega - n\omega_b)$$

where $\boxed{\omega_b = \frac{2\pi}{T_b}}$

9) $H_{RC}(f) = \dfrac{1}{1 + j2\pi f}$ $\qquad H_{diff}(f) = j2\pi f$

$$H(f) = H_{RC}\,H_{diff} = \frac{j2\pi f}{1 + j2\pi f}$$

$$S_y(f) = |H(f)|^2 S_x(f) = \left(1 - \frac{1}{1 + 4\pi^2 f^2}\right)\Pi(0.25\pi f)$$

$$P_y = \frac{1}{\pi} - \frac{1}{4}$$