Solution

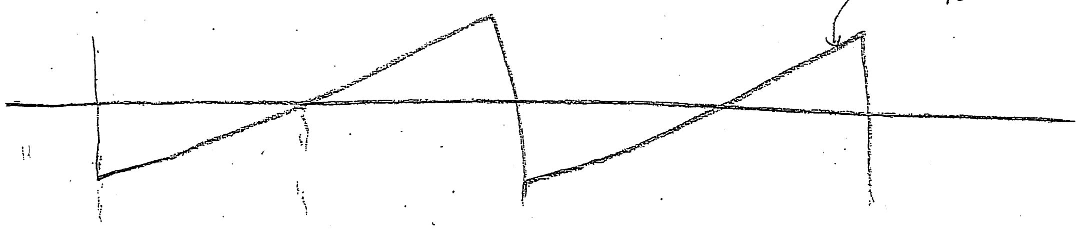
HW-4_Ch4

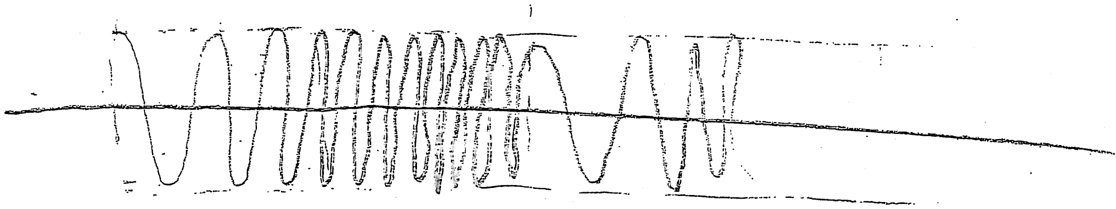
1)

a) For FM $f_{i\_max} = 10^6 + 1000 = 1001$ kHz

$f_{i\_min} = 10^6 - 1000 = 999$ kHz

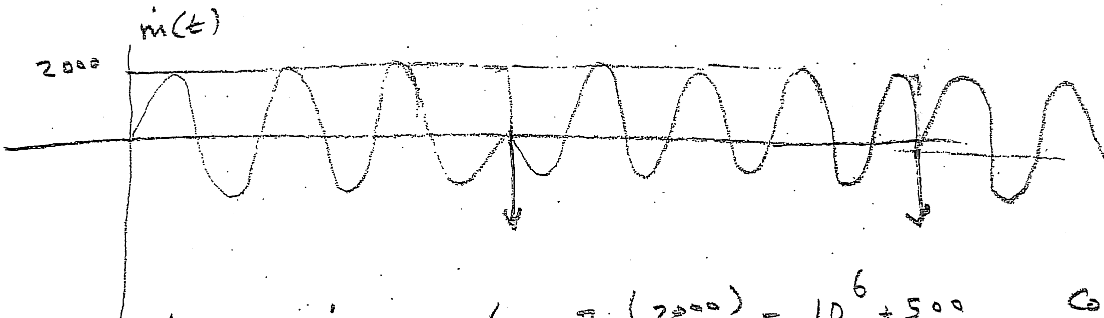
$m(t) = \frac{2}{10^{-3}} t$





$m(t) = \frac{2}{10^{-3}} t$

$\dot{m}(t) = \frac{2}{10^{-3}} = 2000$



$f_i = 10^6 + \frac{k_p \dot{m}_p}{2\pi} = 10^6 + \frac{\pi \cdot (2000)}{2(2\pi)} = 10^6 + 500$ Constant frequency

b) $k_p(2m_p) < 2\pi$ to avoid ambiguity in phase shift at discontinuity point of the signal $m(t)$.

2) a) $\varphi_{PM}(t) = 10\cos(12{,}000\pi t + k_p m(t)) = 10\cos\left(12{,}000\pi t + \overset{k_p}{1000}\, m(t)\right)$

so $1000\pi t + 0.3\pi = 1000\, m(t) \Rightarrow m(t) = \pi t + 0.3 \times 10^{-3}\pi$

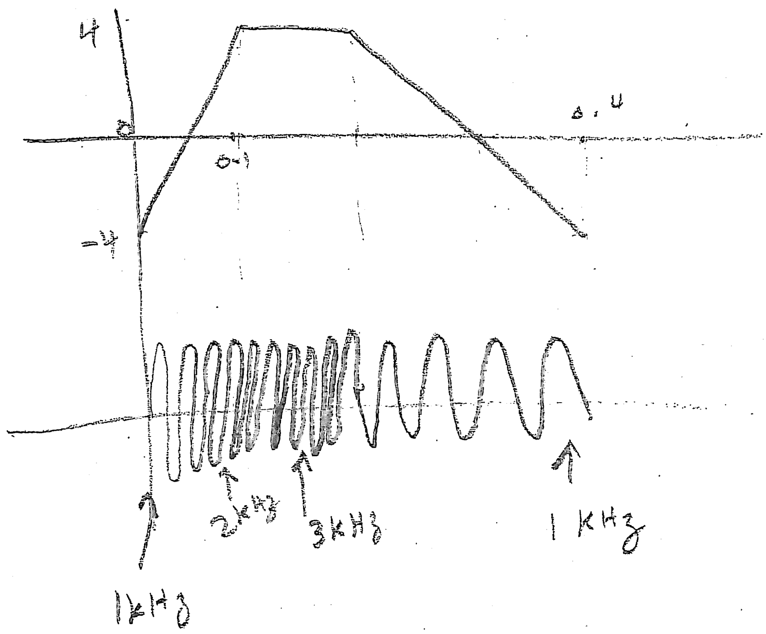
b) $\varphi_{FM}(t) = 10\cos\left(12{,}000\pi t + k_f \int_0^t m(\alpha)\,d\alpha\right)$

$1000\int_0^t m(\alpha)\,d\alpha = 1000\pi t + 0.3\pi \Rightarrow \int_0^t m(\alpha)\,d\alpha = \pi t + 0.3\pi \times 10^{-3}$

$m(t) = \pi$

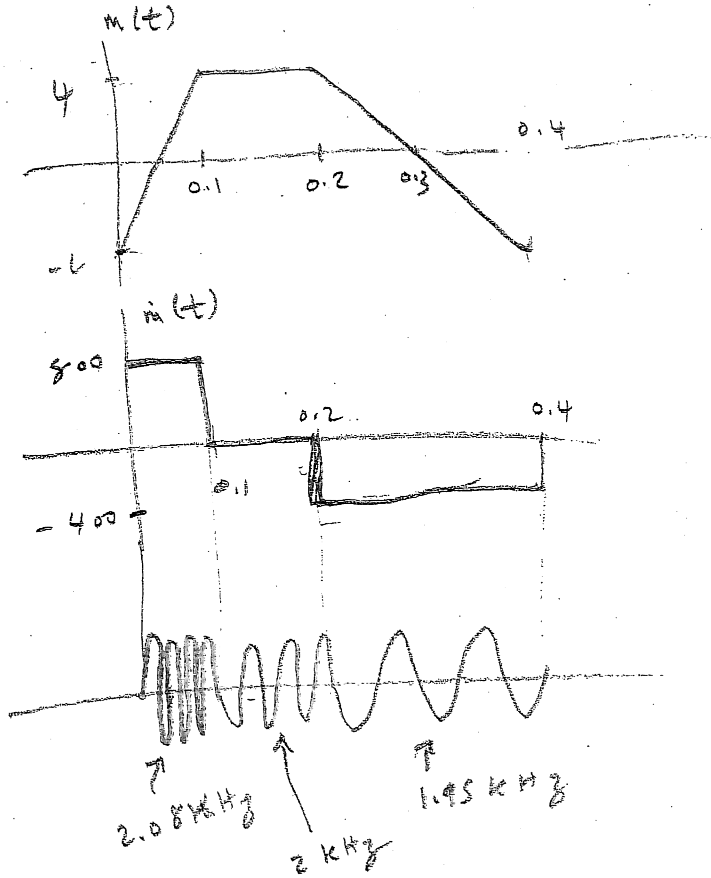
3) For FM $f_i = 2000 + 250\, m(t)$

a) $f_{i_{max}} = 3 \text{ kHz}$ $\qquad f_{i_{min}} = 1 \text{ kHz}$



b) PM

$$f_i = 2\times10^3 + \dot{m}(t)\left(\frac{0.25\pi}{2\pi}\right) = 2\times10^3 + \dot{m}(t)\left(\frac{1}{8}\right)$$



4) $BW = 175\ Hz$

a) $B_{FM} = 2(\Delta f + B) = 2(1000 + 175) = 2350\ Hz$

b) $B_{PM} = 550$

5) a) $\beta = 12.5$

b) $\omega_i(t) = \omega_c - 20{,}000\pi \sin 1000\pi t + 40{,}000\pi \cos 4000\pi t$

$\Delta f = 28575\ Hz$

c) $\theta(t) = 20 \cos 1000\pi t + 10 \sin 4000 t$

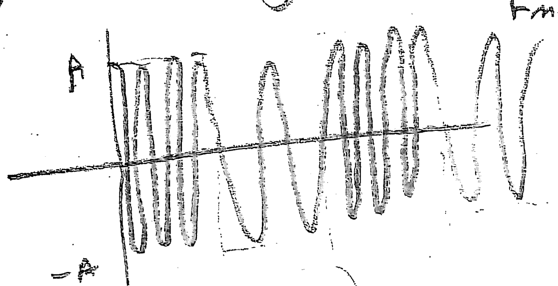
$\Delta \phi = 48.6$

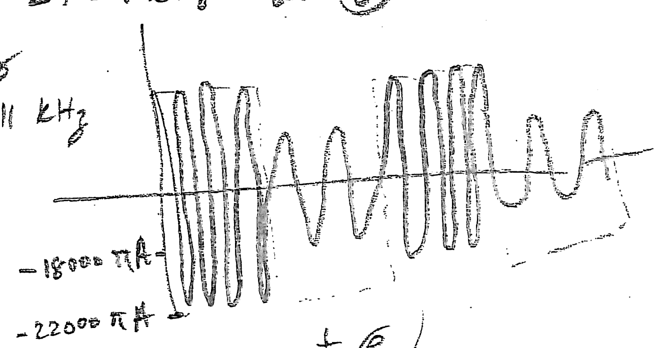
d) $B_{FM} = 2(\Delta f + B) = 2(28575 + 2000) = 61150\ Hz$

6) at (b) $F_c = 10\ kHz$ $\Delta f = 1\ kHz$ at (c)

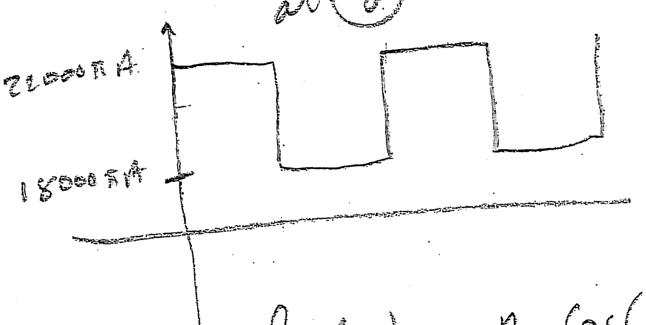
$f_{min} = 9\ kHz$

$f_{max} = 11\ kHz$

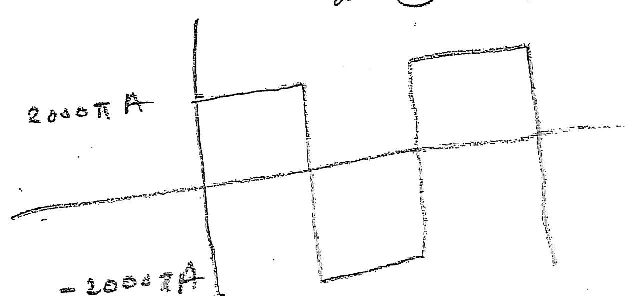




at (d)



at (e)



$\varphi_{FM}(t) = A \cos(2\pi 10000 t \pm 2000\pi t)$

$\dot{\varphi}_{FM}(t) = -(20{,}000\pi \pm 2000\pi) A \sin(20000\pi t \pm 2000\pi t)$

7) The signal bandwidth of $m(t)$ or $\int m(t)$ is the same

a) So the BW of $2\sin(1000\pi t+0.3\pi)-3\cos(2000\pi t)$ is the highest Frequency

$$B = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}$$

$$w_i(t) = w_c + 2000\pi\cos(1000\pi t+0.3\pi) + 6000\pi\sin(2000\pi t)$$

$\Delta w = 8000\pi$ $\quad\quad \Delta f = 4000$ Hz

$$BW = 2(\Delta f + B) = 2(4000+1000) = 10{,}000 \text{ Hz}$$

b) Detector output 2 Volt

c) $$\frac{ds(t)}{dt} = -2\left[10^7\pi - 2000\pi\cos(1000\pi t+0.3\pi) + 6000\pi\sin(2000\pi t)\right]$$

$$\cos\left(10^7\pi t + 2\sin(1000\pi t+0.3\pi) - 3\cos(2000\pi t)\right]$$

detector output $= -2\times10^7\pi + 4000\pi\cos(1000\pi t+0.3\pi) - 12000\pi\sin(2000\pi t)$

d) $m(t) = \dfrac{\text{Detector output part C + DC bloker}}{k_F} = 20\cos(1000\pi t+0.3\pi) - 60\sin(2000\pi t)$

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8) Channels $88 \rightarrow 108$ MHz $\quad f_{IF} = 10.7$ MHz

Range of oscillator $= 98.7 \rightarrow 118.7$ MHz

- The image of the lowest channel $= 88 + 2(10.7) = 109.4$ MHz which is larger than 108
- 108 MHz is the image of $108 - 21.4 = 86.6$ MHz which is lower than 88. So Receiver can not receive a station and its image.

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9) See problem solved in class. $\quad M_1 = 64 \quad M_2 = 32 \quad f_{LO} = 9.8$

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10) a) 3 non-overlapping channels

b) $|\text{possible channel} - \text{neighbor channel}| > 22$ MHz

$$|2.427\times10^9 + i\,5\times10^6 - 2.427\times10^9| > 22\times10^6 \Rightarrow |i| > 4.4$$

$i = 5, 6, \ldots, 9$

Ch $= 9, 10, \ldots, 13$