E.1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}$$

E.2 Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

E.3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \frac{\pi^2}{4}$$

$$Q(x) = \frac{e^{-x^2/2}}{x\sqrt{2\pi}}\left(1 - \frac{1}{x^2} + \frac{1\cdot 3}{x^4} - \frac{1\cdot 3\cdot 5}{x^6} + \cdots\right)$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$\cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

E.4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1} - 1}{r - 1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1} - r^M}{r - 1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1} - b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

E.5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

E.6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x \qquad \cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx}) \qquad \sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx}) \qquad 2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1 \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\cos^2 x - \sin^2 x = \cos 2x \qquad \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$\sin x\sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x\cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x\cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a\cos x + b\sin x = C\cos(x + \theta)$$

in which $C = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

E.7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

$$a_n = D_n + D_{-n} = 2\,\mathrm{Re}\{D_n\}$$
$$b_k = j(D_n - D_{-n}) = -2\,\mathrm{Im}\{D_n\}$$
$$a_n = C_n\cos(\theta_n)$$
$$b_n = -C_n\sin(\theta_n)$$
$$a_0 = D_0 = c_0$$

$$D_n = 0.5(a_n - jb_n)$$
$$D_{-n} = D^*_n = 0.5(a_n + jb_n)$$
$$D_n = 0.5C_n\angle\theta_n = 0.5C_ne^{j\theta_n}$$
$$D_0 = a_0 = C_0$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$
$$\theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$C_n = 2|D_n|$$
$$\theta_n = \angle D_n$$
$$C_0 = a_0 = D_0$$

Dot product: $\langle \mathbf{g}, \mathbf{x} \rangle = ||\mathbf{g}||.||\mathbf{x}||\cos\theta$

Correlation Coefficient

$$\rho = \cos\theta = \frac{\langle g,x \rangle}{||g||\,||x||}$$

$$\rho = \frac{1}{\sqrt{E_gE_x}}\int_{-\infty}^{\infty} g(t)x(t)dt$$

Cross-Correlation

$$\varphi_{zg}(\tau) = \int_{-\infty}^{\infty} z(t)g^*(t-\tau)dt$$

Autocorrelation

$$\varphi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t-\tau)dt$$

Trigonometric Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(2\pi nf_0t) + \sum_{n=1}^{\infty} b_n\sin(2\pi nf_0t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(2\pi nf_0t)dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin(2\pi nf_0t)dt$$

$$x(t) = \underbrace{C_0}_{\text{dc component}} + \sum_{n=1}^{\infty}\underbrace{C_n\cos(2\pi nf_0t + \theta_n)}_{\text{nth harmonic}}$$

$$x(t) = \sum_{n=-\infty}^{\infty} D_ne^{j2\pi f_0nt} \qquad D_n = \frac{1}{T_o}\int_{T_o} x(t)e^{-j2\pi f_0nt}dt$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2}|x(t)|^2dt$$

$$P = C_0^2 + \sum_{n=1}^{\infty} 0.5C_n^2$$

$$P = D_0^2 + 2\sum_{n=1}^{\infty}|D_n|^2$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(\omega) = |X(\omega)|e^{\angle X(\omega)}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$\delta_{T_0}(t) = \frac{1}{T_0}\sum_{n=-\infty}^{n=\infty} e^{jn\omega_0t}$$

TABLE 7.1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

$$\varphi_{AM}(t) = [A+m(t)]\cos\omega_c t$$

Modulation index $\mu = \dfrac{m_p}{A}$

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c+P_s} = \frac{\widetilde{m^2}(t)}{A^2+\widetilde{m^2}(t)}100\%$$

$$H(f) = \begin{cases} 1.e^{-\frac{j\pi}{2}} = -j & f>0 \\ 1.e^{\frac{j\pi}{2}} = j & f<0 \end{cases}$$

$$E = \int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$$

$$t_d(f) = -\frac{1}{2\pi}\frac{d\theta}{df}$$ group delay

TABLE 7.2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t)+x_2(t)$	$X_1(\omega)+X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\dfrac{1}{\|a\|}X\left(\dfrac{\omega}{a}\right)$
Time shifting	$x(t-t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$
Time convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\dfrac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Time differentiation	$\dfrac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\dfrac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$

$$x(t)\cos(\omega_0 t) \Leftrightarrow \frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$$

$$\Psi_g(f) = |G(f)|^2$$

$$\psi_g(\tau) \underset{\text{FT and IFT}}{\Longleftrightarrow} \Psi_g(f)$$

$$\mathcal{R}_g(\tau) \underset{\text{FT and IFT}}{\Longleftrightarrow} S_g(f)$$

$$S_x(f) \rightarrow \boxed{H(f)} \rightarrow S_y(f)$$

$$S_y(f) = |H(f)|^2 S_x(f)$$

$E_g = \int_{-\infty}^{\infty} g^2(t)\,dt$ | $P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g^2(t)\,dt = \lim_{T\to\infty}\frac{E_{g_T}}{T}$

$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)\,dt$ | $\mathcal{R}_g(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g(t)g(t+\tau)\,dt = \lim_{T\to\infty}\frac{\psi_{g_T}(\tau)}{T}$

$\Psi_g(f) = |G(f)|^2$ | $S_g(f) = \lim_{T\to\infty}\frac{|G_T(f)|^2}{T} = \lim_{T\to\infty}\frac{\Psi_{g_T}(f)}{T}$

$\psi_g(\tau) \Longleftrightarrow \Psi_g(f)$ | $\mathcal{R}_g(\tau) \Longleftrightarrow S_g(f)$

$E_g = \int_{-\infty}^{\infty}\Psi_g(f)\,df$ | $P_g = \int_{-\infty}^{\infty} S_g(f)\,df$

$$H(f) = -j\,sgn(f) \qquad h(t) = \frac{1}{\pi t}$$

$$\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

$$\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

$$\varphi_{QAM}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$

$$\Phi_{VSB}(f) = [M(f+f_c)+M(f-f_c)]H_i(f)$$

$$H_o(f) = \frac{1}{H_i(f+f_c)+H_i(f-f_c)} \qquad |f| \le B$$

$$\varphi(t) = A\cos\theta(t) \qquad \omega_i(t) = \frac{d\theta}{dt}$$

$$\varphi_{PM}(t) = A\cos[\omega_C t + k_p m(t)]$$

$$\varphi_{FM}(t) = A\cos\left[\omega_C t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]$$

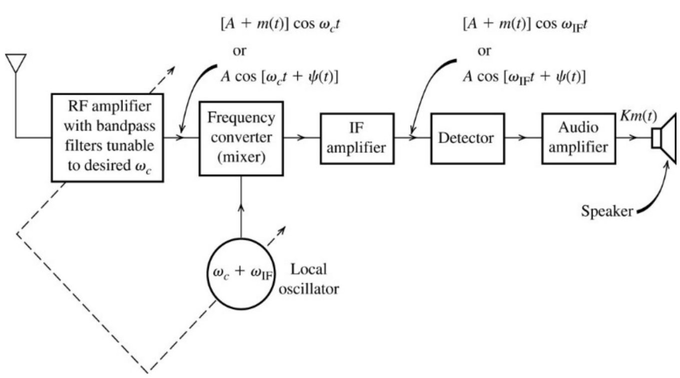
$$B_{FM} = 2(\Delta f + B) = 2B(\beta + 1)\ \text{Hz}$$

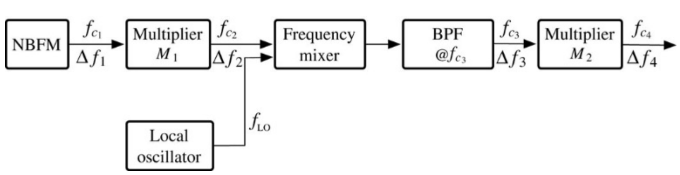
$$\Delta f = \frac{k_f m_p}{2\pi} \qquad \Delta f = k_p \frac{\dot{m}_p}{2\pi} \qquad \beta = \frac{\Delta f}{B}$$

$$\varphi_{NBFM}(t) \approx A[\cos\omega_c t - k_f a(t)\sin\omega_c t]$$

$$a(t) = \int_{-\infty}^{t} m(\alpha)d\alpha$$

$$\varphi_{NBPM}(t) \approx A[\cos\omega_c t - k_p m(t)\sin\omega_c t]$$





Chapter 6

Nyquist rate f_s = 2B

$$\bar{g}(t) = \sum_n g(nT_s)\delta(t - nT_s)$$

$$\bar{G}(f) = \frac{1}{T_s}\sum_{n=-\infty}^{n=\infty} G(f - nf_s)$$

$$E(f)P(f) = \begin{cases} T_s & |f| \le B \\ \text{Flexible} & B < |f| < (f_s - B) \\ 0 & |f| > f_s - B \end{cases}$$

$\Delta v = 2m_p/L \qquad L = 2^n$

$$\widetilde{q^2} = \frac{1}{\Delta v}\int_{-\Delta v/2}^{\Delta v/2} q^2 dq = \frac{(\Delta v)^2}{12} = \frac{m_p^2}{3L^2}$$

$$\frac{S_0}{N_0} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2}$$

$$y = \frac{1}{\ln(1+\mu)}\ln\left(1 + \frac{\mu m}{m_p}\right)$$

$$0 \le \frac{m}{m_p} \le 1$$

$d[k] = m[k] - m[k\text{-}1]$

$$d[k] = m[k] - \widehat{m}[k]$$

$$m_q[k] = m_q[k-2] + d_q[k] + d_q[k-1]$$

$$Ef_s > |\dot{m}(t)|$$

Chapter 8

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A)P(B)$$

$$P(A/B) = P(A)$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$p(k \text{ success in } n \text{ trials}) = \binom{n}{k} p^k (1-p)^{n-k}$

$$P(B) = \sum_{i=1}^{n} P(B/A_i)P(A_i)$$

$$P(A_j/B) = \frac{P(B/A_j)P(A_j)}{P(B)} = \frac{P(B/A_j)P(A_j)}{\sum_{i=1}^{n} P(B/A_i)P(A_i)}$$

$$\int_{-\infty}^{\infty} p_{\mathrm{x}}(x)dx = 1$$

$$P(x_1 < x \le x_2) = \int_{x_1}^{x_2} p_{\mathrm{x}}(x)dx = F_{\mathrm{x}}(x_2) - F_{\mathrm{x}}(x_1)$$

$$p_{\mathrm{x}}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}$$

$$F_{\mathrm{x}}(x) = P(\mathrm{x} \le x) = 1 - Q\left(\frac{x-m}{\sigma}\right)$$

$$\bar{\mathrm{x}} = E[x] = \sum_{i=1}^{n} x_i P_{\mathrm{x}}(x_i)$$

$$\bar{\mathrm{x}} = E[x] = \int_{-\infty}^{\infty} x p_{\mathrm{x}}(x)dx$$

$$\overline{g(x)} = \sum_{i=1}^{n} g(x_i) P_{\mathrm{x}}(x_i)$$

$$\overline{g(x)} = \int_{-\infty}^{\infty} g(x) p_{\mathrm{x}}(x)dx$$

$$\overline{\mathrm{x}+\mathrm{y}} = \bar{\mathrm{x}} + \bar{\mathrm{y}}$$

$$\overline{g_1(x)g_2(y)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(x)g_2(y)p_{\mathrm{xy}}(x,y)dxdy$$

$$\overline{\mathrm{xy}} = \bar{\mathrm{x}}\ \bar{\mathrm{y}}$$

$$\overline{g_1(x)g_2(x)} = \int_{-\infty}^{\infty} g_1(x)p_{\mathrm{x}}(x)dx \int_{-\infty}^{\infty} g_2(y)p_{\mathrm{y}}(y)dy$$

$$\overline{\mathrm{x}^n} = \int_{-\infty}^{\infty} x^n p_{\mathrm{x}}(x)dx$$

$$\overline{(\mathrm{x}-\bar{\mathrm{x}})^n} = \int_{-\infty}^{\infty} (x-\bar{\mathrm{x}})^n p_{\mathrm{x}}(x)dx$$

$$\sigma_{\mathrm{x}}^2 = \overline{(\mathrm{x}-\bar{\mathrm{x}})^2} = \overline{\mathrm{x}^2} - \bar{\mathrm{x}}^2$$