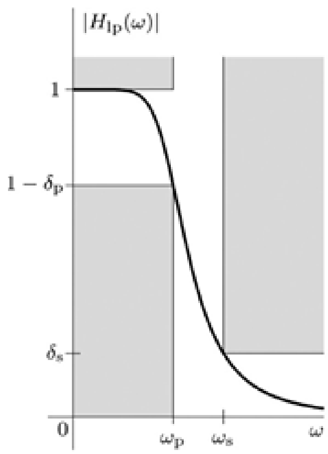
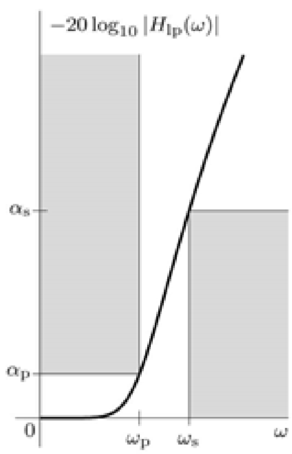
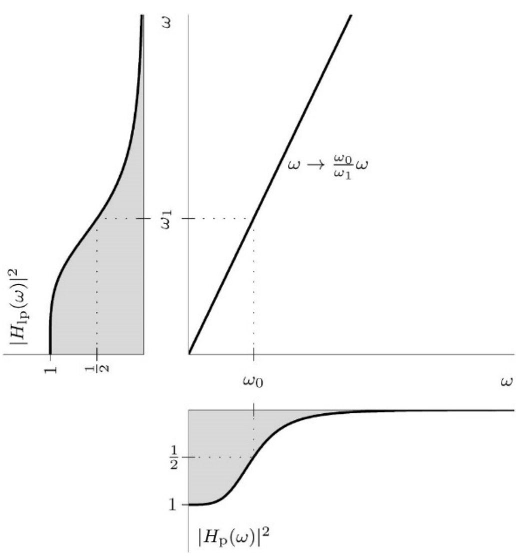
Chapter 2

Window $w(t)$	Main Lobe Width	Rolloff Rate [dB/dec]	Peak Side Lobe Level [dB]
1. Rectangular: $\Pi\left(\frac{t}{T}\right)$	$\frac{4\pi}{T}$	-20	-13.3
2. Triangular (Bartlett): $\Lambda\left(\frac{t}{T}\right)$	$\frac{8\pi}{T}$	-40	-26.5
3. Hann: $\frac{1}{2}\left[1+\cos\left(\frac{2\pi t}{T}\right)\right]\Pi\left(\frac{t}{T}\right)$	$\frac{8\pi}{T}$	-60	-31.5
4. Hamming: $\left[0.54+0.46\cos\left(\frac{2\pi t}{T}\right)\right]\Pi\left(\frac{t}{T}\right)$	$\frac{8\pi}{T}$	-20	-42.7
5. Blackman: $\left[0.42+0.5\cos\left(\frac{2\pi t}{T}\right)+0.08\cos\left(\frac{4\pi t}{T}\right)\right]\Pi\left(\frac{t}{T}\right)$	$\frac{12\pi}{T}$	-60	-58.1
6. Kaiser: $\frac{I_0\left(\alpha\sqrt{1-4\left(\frac{t}{T}\right)^2}\right)}{I_0(\alpha)}\Pi\left(\frac{t}{T}\right)$	varies with α	-20	varies with α

$$\alpha_{\text{p}} = -20\log_{10}(1-\delta_{\text{p}}) \quad \text{and} \quad \alpha_{\text{s}} = -20\log_{10}(\delta_{\text{s}})$$



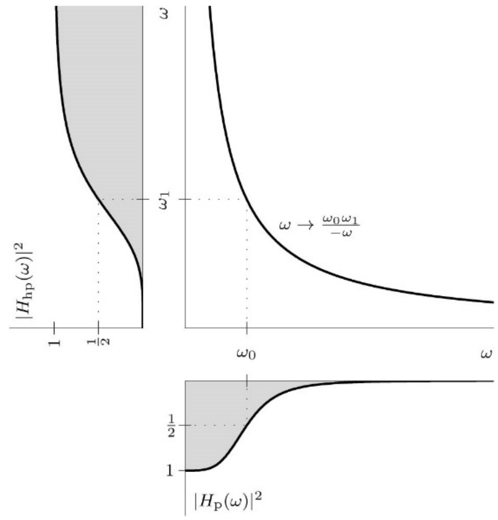




$$s \to \frac{\omega_0}{\omega_1}s \quad \text{or} \quad \omega \to \frac{\omega_0}{\omega_1}\omega$$

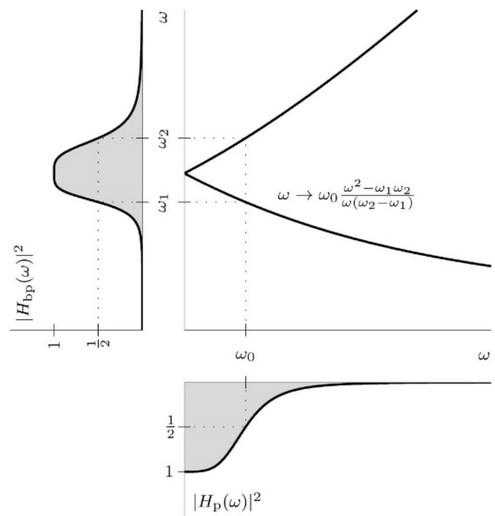
$$H_{\text{lp}}(s) = H_{\text{p}}(s)\big|_{s\to\frac{\omega_0}{\omega_1}s} = H_{\text{p}}\left(\frac{\omega_0}{\omega_1}s\right)$$

$$H_{\text{lp}}(\omega) = H_{\text{p}}(\omega)\big|_{\omega\to\frac{\omega_0}{\omega_1}\omega} = H_{\text{p}}\left(\frac{\omega_0}{\omega_1}\omega\right)$$



$$s \to \frac{\omega_0\omega_1}{s} \quad \text{or} \quad \omega \to \frac{\omega_0\omega_1}{-\omega}$$

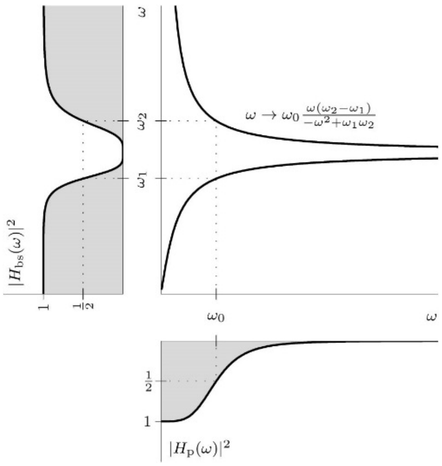
$$H_{\text{hp}}(s) = H_{\text{p}}\left(\frac{\omega_0\omega_1}{s}\right) \qquad H_{\text{hp}}(\omega) = H_{\text{p}}\left(\frac{\omega_0\omega_1}{-\omega}\right)$$



$$s \to \omega_0\frac{s^2+\omega_1\omega_2}{s(\omega_2-\omega_1)} \quad \text{OR} \quad \omega \to \omega_0\frac{\omega^2-\omega_1\omega_2}{\omega(\omega_2-\omega_1)}$$

$$H_{\text{bp}}(s) = H_{\text{p}}\left(\omega_0\frac{s^2+\omega_1\omega_2}{s(\omega_2-\omega_1)}\right)$$

$$H_{\text{bp}}(\omega) = H_{\text{p}}\left(\omega_0\frac{\omega^2-\omega_1\omega_2}{\omega(\omega_2-\omega_1)}\right)$$

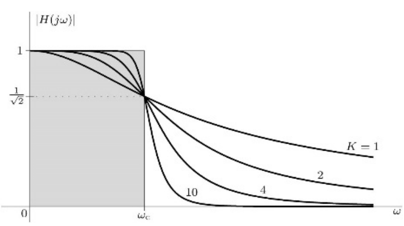


$$s \to \omega_0 \frac{s\,(\omega_2 - \omega_1)}{s^2 + \omega_1\omega_2} \qquad \omega \to \omega_0 \frac{\omega\,(\omega_2 - \omega_1)}{-\omega^2 + \omega_1\omega_2}$$

$$H_{\mathrm{bs}}(s) = H_{\mathrm{p}}\left(\omega_0 \frac{s\,(\omega_2 - \omega_1)}{s^2 + \omega_1\omega_2}\right)$$

$$H_{\mathrm{bs}}(\omega) = H_{\mathrm{p}}\left(\omega_0 \frac{\omega\,(\omega_2 - \omega_1)}{-\omega^2 + \omega_1\omega_2}\right)$$

Butterworth Filters



$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2K}}} \qquad H(s) = \frac{\omega_c^K}{\prod_{k=1}^{K}(s - p_k)}$$

$$p_k = j\omega_c e^{\frac{j\pi}{2K}(2k-1)} \qquad k = 1, 2, 3, \ldots, 2K$$

$$p_k = -\omega_c \sin\left[\frac{\pi(2k-1)}{2K}\right] + j\omega_c \cos\left[\frac{\pi(2k-1)}{2K}\right]$$

K	a_7	a_6	a_5	a_4	a_3	a_2	a_1
2							1.414214
3						2.000000	2.000000
4					2.613126	3.414214	2.613126
5				3.236068	5.236068	5.236068	3.236068
6			3.863703	7.464102	9.141620	7.464102	3.863703
7		4.493959	10.097835	14.591794	14.591794	10.097835	4.493959
8	5.125831	13.137071	21.846151	25.688356	21.846151	13.137071	5.125831

K	Normalized Butterworth Polynomials, factored form
1	$s+1$
2	$s^2+1.414214s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765367s+1)(s^2+1.847759s+1)$
5	$(s+1)(s^2+0.618034s+1)(s^2+1.618034s+1)$
6	$(s^2+0.517638s+1)(s^2+1.414214s+1)(s^2+1.931852s+1)$
7	$(s+1)(s^2+0.445042s+1)(s^2+1.246980s+1)(s^2+1.801938s+1)$
8	$(s^2+0.390181s+1)(s^2+1.111140s+1)(s^2+1.662939s+1)(s^2+1.961571s+1)$

$$\alpha_{\mathrm{p}} = -20\log_{10}|H(j\omega_{\mathrm{p}})| = 10\log_{10}\left[1 + \left(\frac{\omega_{\mathrm{p}}}{\omega_{\mathrm{c}}}\right)^{2K}\right]$$

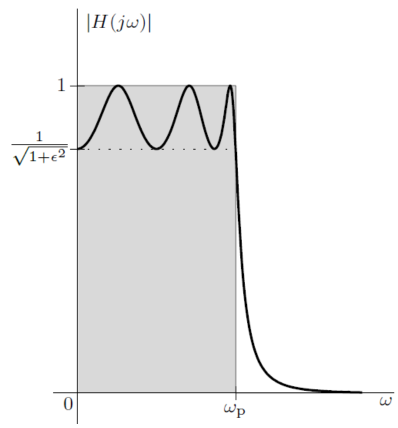
$$\alpha_{\mathrm{s}} = -20\log_{10}|H(j\omega_{\mathrm{s}})| = 10\log_{10}\left[1 + \left(\frac{\omega_{\mathrm{s}}}{\omega_{\mathrm{c}}}\right)^{2K}\right]$$

$$K = \left\lceil \frac{\log\left[\left(10^{\alpha_{\mathrm{s}}/10} - 1\right) / \left(10^{\alpha_{\mathrm{p}}/10} - 1\right)\right]}{2\log(\omega_{\mathrm{s}}/\omega_{\mathrm{p}})} \right\rceil$$

$$\frac{\omega_{\mathrm{p}}}{\left(10^{\alpha_{\mathrm{p}}/10} - 1\right)^{1/2K}} \le \omega_{\mathrm{c}} \le \frac{\omega_{\mathrm{s}}}{\left(10^{\alpha_{\mathrm{s}}/10} - 1\right)^{1/2K}}$$

Chebyshev Filters

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 {C_K}^2\left(\frac{\omega}{\omega_{\mathrm{p}}}\right)}}$$



$$C_K(x) = \cos\left[K\cos^{-1}(x)\right]$$
$$C_K(x) = \cosh\left[K\cosh^{-1}(x)\right]$$
$$C_0(x) = 1 \qquad C_1(x) = x$$

$$\alpha_P = 10\log_{10}(1 + \epsilon^2) \qquad \epsilon^2 = 10^{\alpha_P/10} - 1$$

$$|H(j0)| = \begin{cases} 1 & K \text{ odd} \\ \frac{1}{\sqrt{1+\epsilon^2}} & K \text{ even} \end{cases} \quad \text{and} \quad |H(j\omega_{\mathrm{p}})| = \frac{1}{\sqrt{1+\epsilon^2}}$$

$$\alpha_{\mathrm{s}} = -20\log_{10}|H(j\omega_{\mathrm{s}})| = 10\log_{10}\left[1 + \epsilon^2 {C_K}^2\left(\frac{\omega_{\mathrm{s}}}{\omega_{\mathrm{p}}}\right)\right]$$

$$K = \left\lceil \frac{\cosh^{-1}\sqrt{\left(10^{\alpha_{\mathrm{s}}/10} - 1\right) / \left(10^{\alpha_{\mathrm{p}}/10} - 1\right)}}{\cosh^{-1}(\omega_{\mathrm{s}}/\omega_{\mathrm{p}})} \right\rceil$$

$$p_k = -\omega_{\mathrm{p}}\sinh\left[\frac{1}{K}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]\sin\left[\frac{\pi(2k-1)}{2K}\right] + j\omega_{\mathrm{p}}\cosh\left[\frac{1}{K}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right]\cos\left[\frac{\pi(2k-1)}{2K}\right] \qquad k = 1, 2, \ldots, K$$

Coefficients of normalized Chebyshev denominator poly.

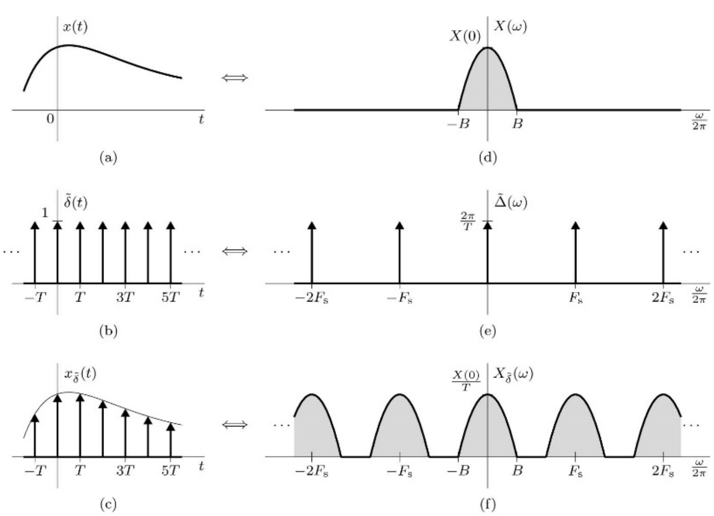
$$s^K + a_{K-1}s^{K-1} + \cdots + a_1 s + a_0$$

K	a_6	a_5	a_4	a_3	a_2	a_1	a_0
1	0.1 dB of ripple						6.552203
2	$(\alpha_p = 0.1)$					2.372356	3.314037
3					1.938811	2.629495	1.638051
4				1.803773	2.626798	2.025501	0.828509
5			1.743963	2.770704	2.396959	1.435558	0.409513
6		1.712166	2.965756	2.779050	2.047841	0.901760	0.207127
7	1.693224	3.183504	3.169246	2.705144	1.482934	0.561786	0.102378
1	0.5 dB of ripple						2.862775
2	$(\alpha_p = 0.5)$					1.425625	1.516203
3					1.252913	1.534895	0.715694
4				1.197386	1.716866	1.025455	0.379051
5			1.172491	1.937367	1.309575	0.752518	0.178923
6		1.159176	2.171845	1.589764	1.171861	0.432367	0.094763
7	1.151218	2.412651	1.869408	1.647903	0.755651	0.282072	0.044731
1	1 dB of ripple						1.965227
2	$(\alpha_p = 1)$					1.097734	1.102510
3					0.988341	1.238409	0.491307
4				0.952811	1.453925	0.742619	0.275628
5			0.936820	1.688816	0.974396	0.580534	0.122827
6		0.928251	1.930825	1.202140	0.939346	0.307081	0.068907
7	0.923123	2.176078	1.428794	1.357545	0.548620	0.213671	0.030707

1	2 dB of ripple						1.307560
2	$(\alpha_p = 2)$					0.803816	0.823060
3					0.737822	1.022190	0.326890
4				0.716215	1.256482	0.516798	0.205765
5			0.706461	1.499543	0.693477	0.459349	0.081723
6		0.701226	1.745859	0.867015	0.771462	0.210271	0.051441
7	0.698091	1.993665	1.039546	1.144597	0.382638	0.166126	0.020431
1	3 dB of ripple						1.002377
2	$(\alpha_p = 3)$					0.644900	0.707948
3					0.597240	0.928348	0.250594
4				0.581580	1.169118	0.404768	0.176987
5			0.574500	1.415025	0.548937	0.407966	0.062649
6		0.570698	1.662848	0.690610	0.699098	0.163430	0.044247
7	0.568420	1.911551	0.831441	1.051845	0.300017	0.146153	0.015662

Chapter 3

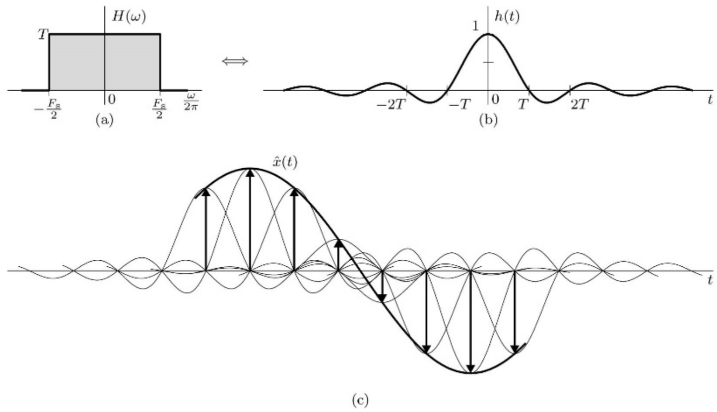
Nyqist sampling rate ≥ 2 Bandwidth of the signal



$$x_{\tilde{\delta}}(t) = x(t)\sum_{n=-\infty}^{n=\infty} \delta(t - nT)$$

$$\tilde{\delta}(t) = \frac{1}{T}[1 + 2\cos(\omega_s t) + 2\cos(2\omega_s t) + \cdots]$$

$$X_{\tilde{\delta}}(\omega) = \frac{1}{T}\sum_{n=-\infty}^{n=\infty} X(\omega - k\omega_s)$$



$$h(t) = sinc(t/T) \qquad \hat{x}(t) = \sum_{k=-\infty}^{\infty} x(nT)h(t - nT)$$

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(nT) sinc\left(\frac{t - nT}{T}\right)$$

$$H(\omega) = T\,\text{sinc}\left(\frac{\omega T}{2\pi}\right)$$ zero-order Hold Filter

$$f_a = \langle f_0 + F_s/2 \rangle_{F_s} - F_s/2$$ apparent frequency

Sampling Bandpass Signal

$$KF_s < 2f_1 \quad \text{and} \quad (K+1)F_s > 2f_2$$

$$K_{\max} = \left\lfloor \frac{f_2}{B} \right\rfloor - 1$$

$$\frac{2}{K+1}\left(\frac{f_2}{B}\right) < \frac{F_s}{B} < \frac{2}{K}\left(\frac{f_2}{B} - 1\right)$$

$$\frac{2f_2}{K+1} < F_s < \frac{2f_1}{K}$$

The spectral Sampling Theorem

The *spectral sampling theorem* states that the spectrum $X(\omega)$ of a signal $x(t)$ time limited to a duration of T_o seconds can be reconstructed from the samples of $X(\omega)$ taken at a rate R samples/Hz, where $R \geq T_o$ (the signal width or duration) is in seconds. The frequency resolution $f_o = 1/T_o$.

Analog to Digital Conversion

$\Delta = 2 V_{ref} / L$ $\qquad$ $L = 2^B$

Quantization error = $\Delta/2$

Energy of Quantization error $E_\mathrm{q} = \frac{V_\mathrm{ref}^2/3}{4^B}$

Quantization Converter

Rounding Asymmetric: $x_\mathrm{q} = \frac{V_\mathrm{ref}}{2^{B-1}} \left\lfloor \frac{x}{V_\mathrm{ref}} 2^{B-1} + \frac{1}{2} \right\rfloor$

Truncating Asymmetric: $x_\mathrm{q} = \frac{V_\mathrm{ref}}{2^{B-1}} \left\lfloor \frac{x}{V_\mathrm{ref}} 2^{B-1} \right\rfloor$

Rounding Symmetric: $x_\mathrm{q} = \frac{V_\mathrm{ref}}{2^{B-1}} \left(\left\lfloor \frac{x}{V_\mathrm{ref}} 2^{B-1} \right\rfloor + \frac{1}{2} \right)$

Truncating Symmetric: $x_\mathrm{q} = \frac{V_\mathrm{ref}}{2^{B-1}} \left(\left\lfloor \frac{x}{V_\mathrm{ref}} 2^{B-1} - \frac{1}{2} \right\rfloor + \frac{1}{2} \right)$

Two's Complement

$$x_\mathrm{q} = V_\mathrm{ref} \left(-c_{B-1} 2^0 + c_{B-2} 2^{-1} + \cdots + c_1 2^{-(B-2)} + c_0 2^{-(B-1)} \right)$$

Offset Binary

$$x_\mathrm{q} = V_\mathrm{ref} \left(-1 + c_{B-1} 2^0 + c_{B-2} 2^{-1} + \cdots + c_1 2^{-(B-2)} + c_0 2^{-(B-1)} \right)$$

μ-law Compression

$$x_\mu = \frac{V_\mathrm{ref}\,\mathrm{sgn}(x)}{\ln(1+\mu)} \ln(1 + \mu |x| / V_\mathrm{ref})$$

Chapter 4

$$x[n]\delta[n-m] = x[m]\delta[n-m]$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

$$\delta[n] = u[n] - u[n-1]$$

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

Sampling

$e^{st} \rightarrow e^{snT} = e^{(\sigma+j\omega)Tn} = e^{\sigma Tn}\, e^{j\omega Tn} = (e^{\sigma T})^n (e^{j\omega T})^n = |z|^n e^{j\Omega n}$

apparent frequencies due to aliasing

$$\Omega_\mathrm{a} = \langle \Omega + \pi \rangle_{2\pi} - \pi$$

Cos(Ωn) = cos([Ω+2πk]n)

$$x[n] = \underbrace{\frac{x[n] + x^*[n]}{2}}_{\mathrm{Re}\{x[n]\}} + j \underbrace{\left(\frac{x[n] - x^*[n]}{2j} \right)}_{\mathrm{Im}\{x[n]\}}$$

$$x[n] = \underbrace{\frac{x[n] + x[-n]}{2}}_{x_\mathrm{e}[n]} + \underbrace{\frac{x[n] - x[-n]}{2}}_{x_\mathrm{o}[n]}$$

$$x[n] = \underbrace{\frac{x[n] + x^*[-n]}{2}}_{x_\mathrm{cs}[n]} + \underbrace{\frac{x[n] - x^*[-n]}{2}}_{x_\mathrm{ca}[n]}$$

Fundamental period N_o

$\cos(\Omega n) = \cos(2\pi \frac{f}{F_s} n) = \cos(2\pi \frac{m}{N_o} n)$

m = number of cycles in one fundamental period N_o

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad x[n] \to 0 \text{ as } |n| \to \infty$$

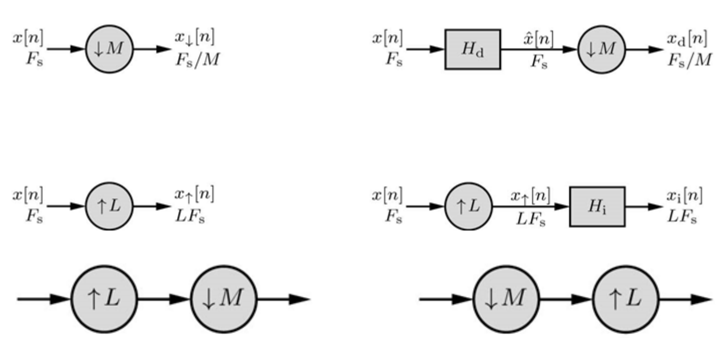
$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$\left. \frac{d^K}{dt^K} y(t) \right|_{t=nT} \approx \frac{1}{T^K} \sum_{k=0}^{K} (-1)^k \binom{K}{k} y[n-k].$$

$$a_0 y[n+K] + a_1 y[n+K-1] + \cdots + a_{K-1} y[n+1] + a_K y[n] = b_0 x[n+L] + b_1 x[n+L-1] + \cdots + b_{L-1} x[n+1] + b_L x[n]$$

$$\sum_{k=0}^{K} a_k y[n+K-k] = \sum_{l=0}^{L} b_l x[n+L-l]$$

$$\underbrace{y[n]}_{\text{output}} = \underbrace{\sum_{m=-\infty}^{\infty}}_{\text{sum}} \underbrace{x[m]}_{\text{scaled}} \underbrace{h[n-m]}_{\text{shifted impulse responses}}$$



Chapter 5

$E\{x[n]\} \equiv x[n+1]$ $\quad E^2\{x[n]\} \equiv x[n+2]$

$E^{-1}\{x[n]\} \equiv x[n-1]$ $\quad E^{-2}\{x[n]\} \equiv x[n-2]$

Zero Input Response

$$y[n+K] + a_1 y[n+K-1] + \cdots + a_{K-1} y[n+1] + a_K y[n] = 0$$

$$(\gamma^K + a_1 \gamma^{K-1} + \cdots + a_{K-1}\gamma + a_K) = A(\gamma) = 0$$

$$(\gamma - \gamma_1)(\gamma - \gamma_2)\cdots(\gamma - \gamma_K) = A(\gamma) = 0$$

$$y[n] = c_1\gamma_1^n + c_2\gamma_2^n + \cdots + c_K\gamma_K^n$$

If two roots are equal then $y[n] = c_1\gamma^n + c_2\gamma^n$

For complex roots $\gamma = |\gamma|e^{j\beta}$ and $\gamma^* = |\gamma|e^{-j\beta}$

$$y[n] = \frac{c}{2}|\gamma|^n \left(e^{j(\beta n+\theta)} + e^{-j(\beta n+\theta)}\right)$$
$$= c|\gamma|^n \cos(\beta n + \theta)$$

The unit impulse response h[n]

$$y[n+K] + a_1 y[n+(K-1)] + \cdots + a_{K-1} y[n+1] + a_K y[n] =$$
$$b_0 x[n+K] + b_1 x[n+(K-1)] + \cdots + b_{K-1} x[n+1] + b_K x[n]$$

$$h[n] = \frac{b_K}{a_K}\delta[n] + y_c[n]u[n]$$

When there are R zero roots then

$$h[n] = A_0\delta[n] + A_1\delta[n-1] + \cdots + A_R\delta[n-|R] + y_c[n]u[n]$$

Convolution

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

1.	$\sum_{m=p}^{n} r^m = \frac{r^p - r^{n+1}}{1-r}$	$r \neq 1$
2.	$\sum_{m=0}^{n} m = \frac{n(n+1)}{2}$	
3.	$\sum_{m=0}^{n} m^2 = \frac{n(n+1)(2n+1)}{6}$	
4.	$\sum_{m=0}^{n} m r^m = \frac{r + [n(r-1)-1]r^{n+1}}{(r-1)^2}$	$r \neq 1$
5.	$\sum_{m=0}^{n} m^2 r^m = \frac{r[(1+r)(1-r^n) - 2n(1-r)r^n - n^2(1-r)^2 r^n]}{(r-1)^3}$	$r \neq 1$

$$x[n] * h[n] = h[n] * x[n]$$

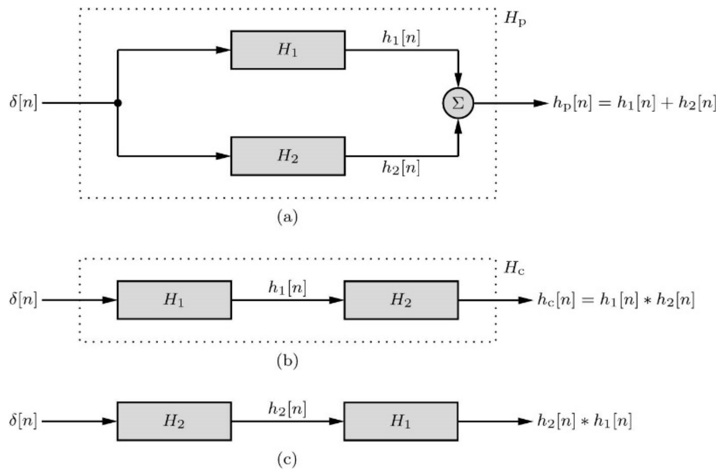
$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

if $x[n] * h[n] = y[n]$ then $x[n-m] * h[n-p] = y[n-m-p]$

$$x[n] * \delta[n] = x[n]$$

if $x[n]$ has width W_x (length L_x) and $h[n]$ has width W_h (length L_h) then the width of the convolution $y[n] = x[n] * h[n]$ is $W_y = W_x + W_h$ (length $L_y = L_x + L_h - 1$)



$$y[n] = h[n] * z^n = \sum_{m=-\infty}^{\infty} h[m] z^{n-m}$$

$$H(z) = \sum_{m=-\infty}^{\infty} h[m] z^{-m}$$

$$y[n] = H(z)z^n = H(z)x[n]\big|_{x[n]=z^n}$$

Finding H(z) from the difference equation

$$A(E)\{y[n]\} = B(E)\{x[n]\}$$

$$H(z) = \frac{B(z)}{A(z)}$$

Finding H(jΩ) from H(z) by replacing z with $e^{j\Omega}$

$$H(e^{j\Omega}) = \sum_{m=-\infty}^{\infty} h[m] e^{-j\Omega m}$$

Total Response can be expressed as

$$y[n] = \underbrace{\left[-1.26(4)^{-n} + 0.444(-0.2)^n + 5.82(0.8)^n\right] u[n]}_{\text{zero-state response}} + \underbrace{0.2(-0.2)^n + 0.8(0.8)^n}_{\text{zero-input response}}$$

$$y[n] = \underbrace{-1.26(4)^{-n}}_{\text{forced response}} + \underbrace{0.644(-0.2)^n + 6.62(0.8)^n}_{\text{natural response}}$$

System Stability

1. A causal LTID system is asymptotically stable if and only if all the characteristic roots are inside the unit circle. The roots may be simple or repeated.

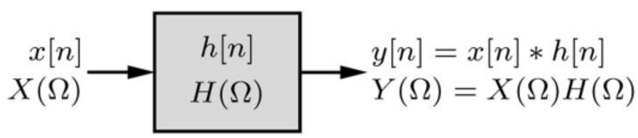
2. A causal LTID system is marginally stable if and only if there are no roots outside the unit circle, and there are non-repeated roots on the unit circle.
3. A causal LTID system is unstable if and only if at least one root is outside the unit circle or there are repeated roots on the unit circle or both.

Chapter 6

DTFT

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

IDFT $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}d\Omega$

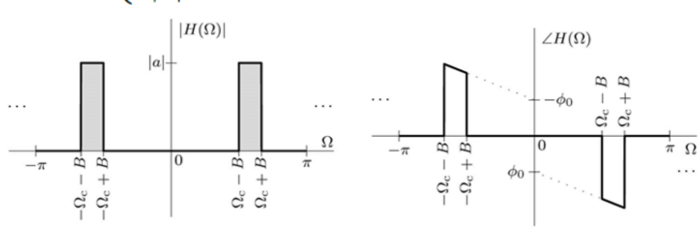


$e^{j\Omega_o n}$ $\qquad$ $H(\Omega_o)e^{j\Omega_o n}$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(\Omega)|^2\, d\Omega$$

Distortionless Transmission in Bandpass Systems

$$H(\Omega) = \begin{cases} |a|e^{j(\phi_0 - \Omega n_g)} & \Omega_c - B \le \Omega \le \Omega_c + B \\ |a|e^{j(-\phi_0 - \Omega n_g)} & -\Omega_c - B \le \Omega \le -\Omega_c + B \end{cases}$$



Group delay

$$n_g(\Omega) = -\frac{d}{d\Omega}\angle H(\Omega)$$

Modulation

$$x[n]\cos(\Omega_0 n) \Longleftrightarrow \frac{1}{2}\left[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)\right]$$

Connection between DTFT $X(\Omega)$ and CTFT $X_c(\omega)$

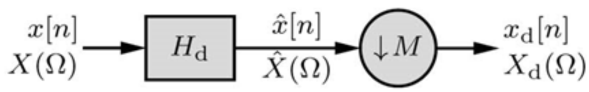
$$X(\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(\frac{\Omega - 2\pi k}{T}\right)$$

Connection between downsampled DTFT $X_\downarrow(\Omega)$ by a factor M and CTFT $X_c(\omega)$

$$X_\downarrow(\Omega) = \frac{1}{TM}\sum_{k=-\infty}^{\infty} X_c\left(\frac{\Omega - 2\pi k}{TM}\right)$$

Connection between downsampled DTFT $X_\downarrow(\Omega)$ by a factor M and the original DTFT $X(\Omega)$

$$X_\downarrow(\Omega) = \frac{1}{M}\sum_{m=0}^{M-1} X\left(\frac{\Omega - 2\pi m}{M}\right)$$



Decimation

$$X_d(\Omega) = \frac{1}{M}\sum_{m=0}^{M-1} X\left(\frac{\Omega - 2\pi m}{M}\right) H_d\left(\frac{\Omega - 2\pi m}{M}\right)$$

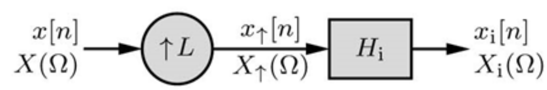
Connection between upsampling DTFT $X_\uparrow(\Omega)$ by a factor L and the original DTFT $X(\Omega)$

$$X_\uparrow(\Omega) = X(L\Omega)$$

Connection between upsampling and ideal interpolated DTFT $X_i(\Omega)$ by a factor L and the original CTFT $X_c(\omega)$

$$X_i(\Omega) = \frac{L}{T}\sum_{k=-\infty}^{\infty} X_c\left(\frac{L[\Omega - 2\pi k]}{T}\right)$$

Connection between upsampling and interpolated DTFT $X_i(\Omega)$ by a factor L and the original DTFT $X(\Omega)$

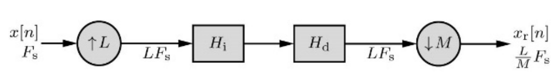


Interpolation

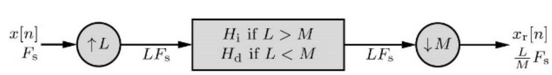
$$X_i(\Omega) = X_\uparrow(\Omega)H_i(\Omega)$$

$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k]h_i[n - kL]$$

Fractional Sampling Rate Conversion



(a)



(b)

- Upsampling by L followed by downsampling by M changes the overall sampling rate by a fractional amount L/M.
- The two lowpass filters H_i and H_d, being in cascade, can be replaced by a single lowpass filter of cutoff frequency π/L or π/M, whichever is lower.
- Preferable to do upsampling prior to downsampling to avoid loss of information.

Chapter 7

The z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

The transfer function H(z)

$$y[n+K] + a_1 y[n+(K-1)] + \cdots + a_{K-1}y[n+1] + a_K y[n] = b_0 x[n+K] + b_1 x[n+(K-1)] + \cdots + b_{K-1}x[n+1] + b_K x[n]$$

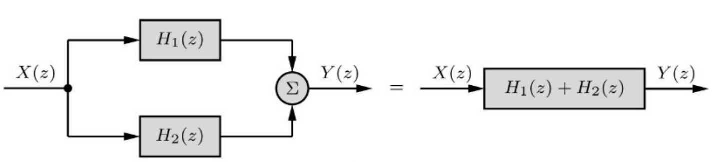
$$H(z) = \frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)} = \frac{b_0 z^K + b_1 z^{K-1} + \cdots + b_{K-1}z + b_K}{z^K + a_1 z^{K-1} + \cdots + a_{K-1}z + a_K}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\mathcal{Z}\{\text{zero-state response}\}}{\mathcal{Z}\{\text{input}\}}$$

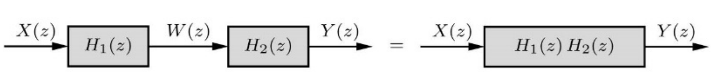
System Stability

- A causal LTID system is asymptotically stable if and only if all the characteristic roots are inside the unit circle. The roots may be simple or repeated.
- A causal LTID system is marginally stable if and only if there are no roots outside the unit circle and there are non-repeated roots on the unit circle.
- A causal LTID system is unstable if and only if at least one root is outside the unit circle, there are repeated roots on the unit circle, or both.

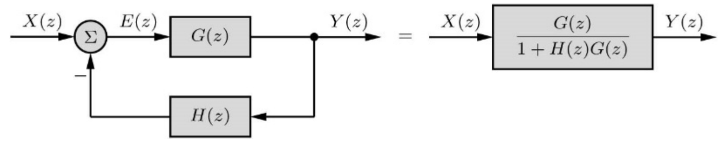
System Block



(a)

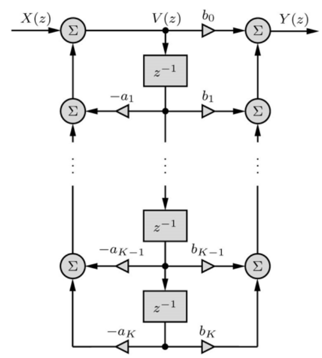


(b)

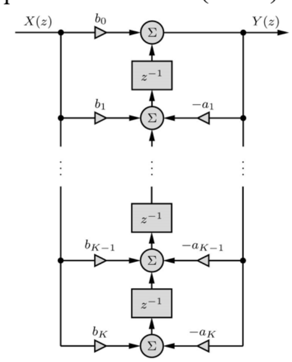


$$H(z) = \frac{b_0 z^L + b_1 z^{L-1} + \cdots + b_{L-1}z + b_L}{z^K + a_1 z^{K-1} + \cdots + a_{K-1}z + a_K}$$

Direct Form II (DFII)



Transpose Direct Form II (TDFII)



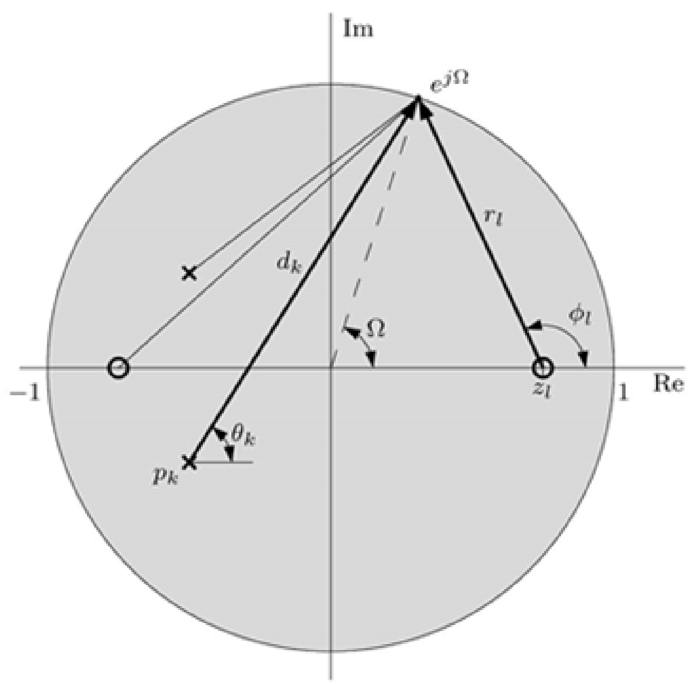
Frequency Response from Pole-zero locations

$$|H(e^{j\Omega})| = |b_0| \frac{r_1 r_2 \cdots r_K}{d_1 d_2 \cdots d_K} = |b_0| \frac{\prod_{l=1}^{K} r_l}{\prod_{k=1}^{K} d_k}$$

$$= |b_0| \frac{\text{product of the distances of zeros to } e^{j\Omega}}{\text{product of the distances of poles to } e^{j\Omega}},$$

$$\angle H(e^{j\Omega}) = \angle b_0 + (\phi_1 + \phi_2 + \cdots \phi_K) - (\theta_1 + \theta_2 + \cdots + \theta_K) = \angle b_0 + \sum_{l=1}^{K} \phi_l - \sum_{k=1}^{K} \theta_k$$

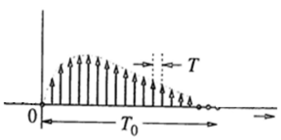
$$= \angle b_0 + \text{sum of zero angles to } e^{j\Omega} - \text{sum of pole angles to } e^{j\Omega}$$



Chapter 9

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\Omega_0 kn} \qquad 0 \le k \le N-1 \qquad\qquad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\Omega_0 kn} \qquad 0 \le n \le N-1$$

$$\Omega_0 = \omega_0 T = 2\pi f_0 T = \frac{2\pi f_0}{f_s} = \frac{2\pi}{N}$$



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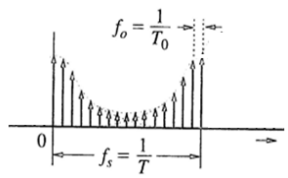


$$f_0 = \frac{1}{T_0} \qquad N = \frac{T_0}{T} \qquad N = \frac{f_s}{f_0} \qquad f_0 = \frac{f_s}{N}$$

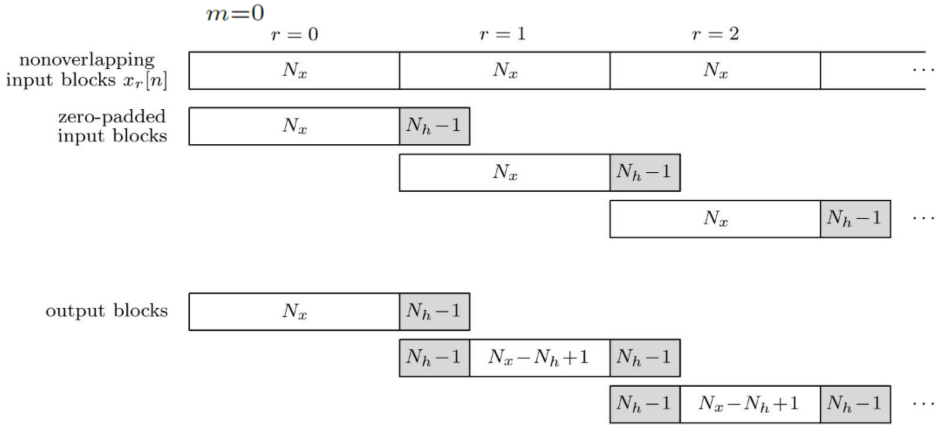
$$X(\Omega) = \frac{1}{N}\sum_{k=0}^{N-1} X[k] \frac{\sin\left(\frac{\Omega N - 2\pi k}{2}\right)}{\sin\left(\frac{\Omega N - 2\pi k}{2N}\right)} e^{-j(\Omega N - 2\pi k)(N-1)/2N}$$

if $x[n] \longleftrightarrow X[k]$, then $x[\langle -n\rangle_N] \longleftrightarrow X[\langle -k\rangle_N] = X[N-k]$

if $x[n] \longleftrightarrow X[k]$, then

$$x[\langle n-m\rangle_N] \longleftrightarrow X[k]e^{-j\Omega_0 km}$$

$$x[n] \circledast h[n] = \sum_{m=0}^{N-1} x[m] h[\langle n-m\rangle_N] \quad \Longleftrightarrow \quad Y(k) = X[k]\,H[k]$$



	$x[n]$	$X(z)$	ROC
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
3.	$\gamma^n u[n]$	$\frac{z}{z-\gamma}$	$\lvert z\rvert > \lvert\gamma\rvert$
4.	$\gamma^{n-1} u[n-1]$	$\frac{1}{z-\gamma}$	$\lvert z\rvert > \lvert\gamma\rvert$
5.	$n\gamma^n u[n]$	$\frac{\gamma z}{(z-\gamma)^2}$	$\lvert z\rvert > \lvert\gamma\rvert$
6.	$n^2\gamma^n u[n]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$	$\lvert z\rvert > \lvert\gamma\rvert$
7.	$\frac{n!}{(n-m)!m!}\gamma^{n-m} u[n]$	$\frac{z}{(z-\gamma)^{m+1}}$	$\lvert z\rvert > \lvert\gamma\rvert$
8.	$\lvert\gamma\rvert^n \cos(\beta n) u[n]$	$\frac{z[z-\lvert\gamma\rvert\cos(\beta)]}{z^2-2\lvert\gamma\rvert\cos(\beta)z+\lvert\gamma\rvert^2}$	$\lvert z\rvert > \lvert\gamma\rvert$
9.	$\lvert\gamma\rvert^n \sin(\beta n) u[n]$	$\frac{z\lvert\gamma\rvert\sin(\beta)}{z^2-2\lvert\gamma\rvert\cos(\beta)z+\lvert\gamma\rvert^2}$	$\lvert z\rvert > \lvert\gamma\rvert$
10.	$\lvert\gamma\rvert^n \cos(\beta n+\theta) u[n]$	$\frac{z[z\cos(\theta)-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-2\lvert\gamma\rvert\cos(\beta)z+\lvert\gamma\rvert^2}$ $= \frac{(0.5e^{j\theta})z}{z-\lvert\gamma\rvert e^{j\beta}} + \frac{(0.5e^{-j\theta})z}{z-\lvert\gamma\rvert e^{-j\beta}}$	$\lvert z\rvert > \lvert\gamma\rvert$
11.	$r\lvert\gamma\rvert^n \cos(\beta n+\theta) u[n]$ $r = \sqrt{\frac{a^2\lvert\gamma\rvert^2+b^2-2abc}{\lvert\gamma\rvert^2-c^2}}$ $\beta = \cos^{-1}\left(\frac{-c}{\lvert\gamma\rvert}\right)$ $\theta = \tan^{-1}\left(\frac{ac-b}{a\sqrt{\lvert\gamma\rvert^2-c^2}}\right)$	$\frac{z(az+b)}{z^2+2cz+\lvert\gamma\rvert^2}$	$\lvert z\rvert > \lvert\gamma\rvert$
12.	$\delta[n-k]$	z^{-k}	$\lvert z\rvert > 0 \quad k>0$ $\lvert z\rvert < \infty \quad k<0$
13.	$-u[-n-1]$	$\frac{z}{z-1}$	$\lvert z\rvert < 1$
14.	$-\gamma^n u[-n-1]$	$\frac{z}{z-\gamma}$	$\lvert z\rvert < \lvert\gamma\rvert$
15.	$-n\gamma^n u[-n-1]$	$\frac{z\gamma}{(z-\gamma)^2}$	$\lvert z\rvert < \lvert\gamma\rvert$

Bilateral z-Transform	Unilateral z-Transform
Synthesis: $x[n] = \frac{1}{2\pi j} \oint X(z)\, z^{n-1}\, dz$	**Synthesis:** $x[n] = \frac{1}{2\pi j} \oint X(z)\, z^{n-1}\, dz$
Analysis: $X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$, ROC: R_x	**Analysis:** $X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}$
Linearity: $ax[n]+by[n] \overset{\mathcal{Z}}{\Longleftrightarrow} aX(z)+bY(z)$, ROC: At least $R_x \cap R_y$	**Linearity:** $ax[n]+by[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} aX(z)+bY(z)$
Complex Conjugation: $x^*[n] \overset{\mathcal{Z}}{\Longleftrightarrow} X^*(z^*)$, ROC: R_x	**Complex Conjugation:** $x^*[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} X^*(z^*)$
Time Reversal: $x[-n] \overset{\mathcal{Z}}{\Longleftrightarrow} X(1/z)$, ROC: $1/R_x$	**Time Reversal:**
Time Shifting: $x[n-m] \overset{\mathcal{Z}}{\Longleftrightarrow} z^{-m}X(z)$, ROC: Almost R_x	**Time Shifting:** If $m>0$: $x[n-m]u[n-m] \overset{\mathcal{Z}_u}{\Longleftrightarrow} z^{-m}X(z)$ (general case given below)
z-Domain Scaling: $\gamma^n x[n] \overset{\mathcal{Z}}{\Longleftrightarrow} X(z/\gamma)$, ROC: $\lvert\gamma\rvert R_x$	**z-Domain Scaling:** $\gamma^n x[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} X(z/\gamma)$
z-Domain Differentiation: $nx[n] \overset{\mathcal{Z}}{\Longleftrightarrow} -z\frac{d}{dz}X(z)$, ROC: R_x	**z-Domain Differentiation:** $nx[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} -z\frac{d}{dz}X(z)$
Time Convolution: $x[n] * y[n] \overset{\mathcal{Z}}{\Longleftrightarrow} X(z)Y(z)$, ROC: At least $R_x \cap R_y$	**Time Convolution:** $x[n] * y[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} X(z)Y(z)$

Unilateral z-Transform Time Shifting, General Case

If $m > 0$: $x[n-m]u[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} z^{-m}X(z) + z^{-m}\sum_{n=1}^{m} x[-n]z^{n}$

If $m < 0$: $x[n-m]u[n] \overset{\mathcal{Z}_u}{\Longleftrightarrow} z^{-m}X(z) - z^{-m}\sum_{n=0}^{-m-1} x[n]z^{-n}$

	$x[n]$	$h[n]$	$x[n] * h[n]$
1.	$x[n]$	$\delta[n-k]$	$x[n-k]$
2.	$u[n]$	$u[n]$	$(n+1)u[n]$
3.	$u[n]$	$n\,u[n]$	$\frac{n(n+1)}{2}u[n]$
4.	$nu[n]$	$nu[n]$	$\frac{n(n-1)(n+1)}{6}u[n]$
5.	$u[n]$	$\gamma^n u[n]$	$\left(\frac{1-\gamma^{n+1}}{1-\gamma}\right)u[n]$
6.	$nu[n]$	$\gamma^n u[n]$	$\left(\frac{\gamma(\gamma^n-1)+n(1-\gamma)}{(1-\gamma)^2}\right)u[n]$
7.	$\gamma^n u[n]$	$\gamma^n u[n]$	$(n+1)\gamma^n u[n]$
8.	$\gamma_1^n u[n]$	$\gamma_2^n u[n]$	$\left(\frac{\gamma_1^{n+1}-\gamma_2^{n+1}}{\gamma_1-\gamma_2}\right)u[n] \qquad \gamma_1 \neq \gamma_2$
9.	$\gamma_1^n u[n]$	$n\gamma_2^n u[n]$	$\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left(\gamma_1^n + \frac{\gamma_2-\gamma_1}{\gamma_1}n\gamma_2^n - \gamma_2^n\right)u[n] \qquad \gamma_1 \neq \gamma_2$
10.	$\gamma_1^n u[n]$	$\gamma_2^n u[-n-1]$	$\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^n u[n] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^n u[-n-1] \qquad \lvert\gamma_2\rvert > \lvert\gamma_1\rvert$
11.	$\lvert\gamma_1\rvert^n u[n]$	$\lvert\gamma_2\rvert^n \cos(\beta n+\theta)u[n]$	$\frac{1}{R}\left[\lvert\gamma_2\rvert^{n+1}\cos[\beta(n+1)+\theta-\phi] - \lvert\gamma_1\rvert^{n+1}\cos(\theta-\phi)\right]u[n]$
			$R = \sqrt{\lvert\gamma_1\rvert^2 - 2\lvert\gamma_1\rvert\lvert\gamma_2\rvert\cos(\beta) + \lvert\gamma_2\rvert^2}$
			$\phi = \tan^{-1}\left[\frac{\lvert\gamma_2\rvert\sin(\beta)}{\lvert\gamma_2\rvert\cos(\beta)-\lvert\gamma_1\rvert}\right]$

Discrete-Time Fourier Transform	Fourier Transform
Synthesis: $x[n] = \frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{j\Omega n}\,d\Omega$	**Synthesis:** $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$
Analysis: $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$	**Analysis:** $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$
Duality:	**Duality:** if $x(t) \Longleftrightarrow X(\omega)$, then $X(t) \Longleftrightarrow 2\pi x(-\omega)$
Linearity: $ax[n] + by[n] \Longleftrightarrow aX(\Omega) + bY(\Omega)$	**Linearity:** $ax(t) + by(t) \Longleftrightarrow aX(\omega) + bY(\omega)$
Complex Conjugation: $x^*[n] \Longleftrightarrow X^*(-\Omega)$	**Complex Conjugation:** $x^*(t) \Longleftrightarrow X^*(-\omega)$
Scaling and Reversal: (see Sec. 6.6) $x[-n] \Longleftrightarrow X(-\Omega)$	**Scaling and Reversal:** $x(at) \Longleftrightarrow \frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$ $x(-t) \Longleftrightarrow X(-\omega)$
Shifting: $x[n-m] \Longleftrightarrow X(\Omega)e^{-j\Omega m}$ $x[n]e^{j\Omega_0 n} \Longleftrightarrow X(\Omega-\Omega_0)$	**Shifting:** $x(t-t_0) \Longleftrightarrow X(\omega)e^{-j\omega t_0}$ $x(t)e^{j\omega_0 t} \Longleftrightarrow X(\omega-\omega_0)$
Differentiation: $-jnx[n] \Longleftrightarrow \frac{d}{d\Omega}X(\Omega)$	**Differentiation:** $\frac{d}{dt}x(t) \Longleftrightarrow j\omega X(\omega)$ $-jtx(t) \Longleftrightarrow \frac{d}{d\omega}X(\omega)$
Time Integration:	**Time Integration:** $\int_{-\infty}^{t} x(\tau)d\tau \Longleftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$
Convolution: $x[n] * y[n] \Longleftrightarrow X(\Omega)Y(\Omega)$ $x[n]y[n] \Longleftrightarrow \frac{1}{2\pi}X(\Omega)\circledast Y(\Omega)$	**Convolution:** $x(t) * y(t) \Longleftrightarrow X(\omega)Y(\omega)$ $x(t)y(t) \Longleftrightarrow \frac{1}{2\pi}X(\omega) * Y(\omega)$
Correlation: $\rho_{x,y}[l] = x[l] * y^*[-l] \Longleftrightarrow X(\Omega)Y^*(\Omega)$	**Correlation:** $\rho_{x,y}(\tau) = x(\tau) * y^*(-\tau) \Longleftrightarrow X(\omega)Y^*(\omega)$
Parseval's: $E_x = \sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(\Omega)\rvert^2\,d\Omega$	**Parseval's:** $E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\,d\omega$

	$x(t)$	$X(\omega)$	
1.	$e^{\lambda t}u(t)$	$\frac{1}{j\omega-\lambda}$	$\mathrm{Re}\{\lambda\} < 0$
2.	$e^{\lambda t}u(-t)$	$-\frac{1}{j\omega-\lambda}$	$\mathrm{Re}\{\lambda\} > 0$
3.	$e^{\lambda\|t\|}$	$\frac{-2\lambda}{\omega^2+\lambda^2}$	$\mathrm{Re}\{\lambda\} < 0$
4.	$t^k e^{\lambda t}u(t)$	$\frac{k!}{(j\omega-\lambda)^{k+1}}$	$\mathrm{Re}\{\lambda\} < 0$
5.	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
6.	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
7.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\mathrm{sinc}\left(\frac{\tau\omega}{2\pi}\right)$	$\tau > 0$
8.	$\frac{B}{\pi}\mathrm{sinc}\left(\frac{Bt}{\pi}\right)$	$\Pi\left(\frac{\omega}{2B}\right)$	$B > 0$
9.	$\Lambda\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$	$\tau > 0$
10.	$\frac{B}{2\pi}\mathrm{sinc}^2\left(\frac{Bt}{2\pi}\right)$	$\Lambda\left(\frac{\omega}{2B}\right)$	$B > 0$
11.	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	$\sigma > 0$
12.	$\delta(t)$	1	
13.	1	$2\pi\delta(\omega)$	
14.	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
15.	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	
16.	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
17.	$\cos(\omega_0 t)$	$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$	
18.	$\sin(\omega_0 t)$	$\frac{\pi}{j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$	
19.	$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]+\frac{j\omega}{\omega_0^2-\omega^2}$	
20.	$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
21.	$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$	$\omega_0 = \frac{2\pi}{T}$

	$x[n]$	$X(\Omega)$ for $-\pi \le \Omega < \pi$	
1.	$\delta[n-k]$	$e^{-jk\Omega}$	Integer k
2.	$\gamma^n u[n]$	$\frac{e^{j\Omega}}{e^{j\Omega}-\gamma}$	$\|\gamma\| < 1$
3.	$-\gamma^n u[-n-1]$	$\frac{e^{j\Omega}}{e^{j\Omega}-\gamma}$	$\|\gamma\| > 1$
4.	$\gamma^{\|n\|}$	$\frac{1-\gamma^2}{1-2\gamma\cos(\Omega)+\gamma^2}$	$\|\gamma\| < 1$
5.	$n\gamma^n u[n]$	$\frac{\gamma e^{j\Omega}}{(e^{j\Omega}-\gamma)^2}$	$\|\gamma\| < 1$
6.	$\|\gamma\|^n \cos(\Omega_0 n+\theta)u[n]$	$\frac{e^{j\Omega}[e^{j\Omega}\cos(\theta)-\|\gamma\|\cos(\Omega_0-\theta)]}{e^{j2\Omega}-2\|\gamma\|\cos(\Omega_0)e^{j\Omega}+\|\gamma\|^2}$	$\|\gamma\| < 1$
7.	$u[n]-u[n-L_x]$	$\frac{\sin(L_x\Omega/2)}{\sin(\Omega/2)}\, e^{-j\Omega(L_x-1)/2}$	
8.	$\frac{B}{\pi}\operatorname{sinc}\left(\frac{Bn}{\pi}\right)$	$\Pi\left(\frac{\Omega}{2B}\right)$	$0 < B \le \pi$
9.	$\frac{B}{2\pi}\operatorname{sinc}^2\left(\frac{Bn}{2\pi}\right)$	$\Lambda\left(\frac{\Omega}{2B}\right)$	$0 < B \le \pi$
10.	1	$2\pi\delta(\Omega)$	
11.	$u[n]$	$\frac{e^{j\Omega}}{e^{j\Omega}-1} + \pi\delta(\Omega)$	
12.	$e^{j\Omega_0 n}$	$2\pi\delta(\Omega-\Omega_0)$	$\|\Omega_0\| < \pi$
13.	$\cos(\Omega_0 n)$	$\pi\left[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)\right]$	$\|\Omega_0\| < \pi$
14.	$\sin(\Omega_0 n)$	$\frac{\pi}{j}\left[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)\right]$	$\|\Omega_0\| < \pi$
15.	$\cos(\Omega_0 n)\, u[n]$	$\frac{e^{j2\Omega}-e^{j\Omega}\cos(\Omega_0)}{e^{j2\Omega}-2\cos(\Omega_0)e^{j\Omega}+1} + \frac{\pi}{2}\left[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)\right]$	$\|\Omega_0\| < \pi$
16.	$\sin(\Omega_0 n)\, u[n]$	$\frac{e^{j\Omega}\sin(\Omega_0)}{e^{j2\Omega}-2\cos(\Omega_0)e^{j\Omega}+1} + \frac{\pi}{2j}\left[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)\right]$	$\|\Omega_0\| < \pi$

	$x[n]$	$X(\Omega)$	
1.	$\delta[n-k]$	$e^{-jk\Omega}$	Integer k
2.	$\gamma^n u[n]$	$\frac{e^{j\Omega}}{e^{j\Omega}-\gamma}$	$\|\gamma\| < 1$
3.	$-\gamma^n u[-n-1]$	$\frac{e^{j\Omega}}{e^{j\Omega}-\gamma}$	$\|\gamma\| > 1$
4.	$\gamma^{\|n\|}$	$\frac{1-\gamma^2}{1-2\gamma\cos(\Omega)+\gamma^2}$	$\|\gamma\| < 1$
5.	$n\gamma^n u[n]$	$\frac{\gamma e^{j\Omega}}{(e^{j\Omega}-\gamma)^2}$	$\|\gamma\| < 1$
6.	$\|\gamma\|^n \cos(\Omega_0 n+\theta)u[n]$	$\frac{e^{j\Omega}[e^{j\Omega}\cos(\theta)-\|\gamma\|\cos(\Omega_0-\theta)]}{e^{j2\Omega}-2\|\gamma\|\cos(\Omega_0)e^{j\Omega}+\|\gamma\|^2}$	$\|\gamma\| < 1$
7.	$u[n]-u[n-L_x]$	$\frac{\sin(L_x\Omega/2)}{\sin(\Omega/2)}\, e^{-j\Omega(L_x-1)/2}$	
8.	$\frac{B}{\pi}\operatorname{sinc}\left(\frac{Bn}{\pi}\right)$	$\sum_{k=-\infty}^{\infty}\Pi\left(\frac{\Omega-2\pi k}{2B}\right)$	
9.	$\frac{B}{2\pi}\operatorname{sinc}^2\left(\frac{Bn}{2\pi}\right)$	$\sum_{k=-\infty}^{\infty}\Lambda\left(\frac{\Omega-2\pi k}{2B}\right)$	
10.	1	$2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$	
11.	$u[n]$	$\frac{e^{j\Omega}}{e^{j\Omega}-1} + \pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$	
12.	$e^{j\Omega_0 n}$	$2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)$	
13.	$\cos(\Omega_0 n)$	$\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)+\delta(\Omega+\Omega_0-2\pi k)$	
14.	$\sin(\Omega_0 n)$	$\frac{\pi}{j}\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)-\delta(\Omega+\Omega_0-2\pi k)$	
15.	$\cos(\Omega_0 n)\, u[n]$	$\frac{e^{j2\Omega}-e^{j\Omega}\cos(\Omega_0)}{e^{j2\Omega}-2\cos(\Omega_0)e^{j\Omega}+1} + \frac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)+\delta(\Omega+\Omega_0-2\pi k)$	
16.	$\sin(\Omega_0 n)\, u[n]$	$\frac{e^{j\Omega}\sin(\Omega_0)}{e^{j2\Omega}-2\cos(\Omega_0)e^{j\Omega}+1} + \frac{\pi}{2j}\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_0-2\pi k)-\delta(\Omega+\Omega_0-2\pi k)$	