ENGR 4333/5333: Digital Signal Processing

HW 11: Ch 9

1) Consider the signal $x[n] = \delta[n] - 2\delta[n-1] + 2\delta[n-2] - \delta[n-3]$.

- **(a)** Compute and plot the 4-point DFT $X[k]$.
- **(b)** Using the fewest number of zeros possible, zero pad $x[n]$ so that the DFT frequency resolution is at least $\Omega_0 = 0.1$. Compute and plot the corresponding DFT $X_{\text{pad}}[k]$.

2) The 8-point DFT of a real signal $x[n]$ is known over $0 \le k \le 4$ to be $X[k] = 1 + 2j\delta[k-2] + \delta[k-4]$.

- **(a)** Determine $X[k]$ for $5 \le k \le 7$.
- **(b)** Determine and plot the corresponding time-domain signal $x[n]$.

3) Consider a length-4 unit amplitude pulse $x[n] = u[n] - u[n-4]$.

- **(a)** Compute and plot the 4-point DFT $X[k]$ of signal $x[n]$.
- **(b)** Use the DFT interpolation formula to compute the DTFT $X(\Omega)$ from the DFT $X[k]$. Plot $X(\Omega)$.
- **(c)** Sample $X(\Omega)$ over $0 \le \Omega < 2\pi$ using 10 equally spaced points to create a signal $X_{10}[k]$. Compute and plot the IDFT of $X_{10}[k]$. Comment on your results.

4) This problem investigates zero padding applied in the frequency domain. Plot each N-point DFT as a function of frequency $f_k = k/N$.

- **a)** In MATLAB, create a vector x that contains one period of the sinusoid $x[n] = \cos(\pi/2\, n)$. Plot the result. How "sinusoidal" does the signal appear?
- **b)** Using the fft command, compute the DFT X of vector x. Plot the magnitude of the DFT coefficients. Do they make sense?
- **c)** Zero pad the DFT vector to a total length of 100 by inserting the appropriate number of zeros in the middle of the vector X. Call this zero-padded DFT sequence Y. Why are zeros inserted in the middle rather than at the end? Take the inverse DFT of Y and plot the result. What similarities exist between the new signal y and the original signal x? What are the differences between x and y? What is the effect of zero padding in the frequency domain? How is this type of zero padding similar to zero padding in the time domain?
- **d)** Derive a general modification to the procedure of zero padding in the frequency domain to ensure that the amplitude of the resulting time-domain signal is left unchanged.
- **e)** Consider one period of a square wave described by the length-8 vector [1, 1, 1, 1, −1, −1, −1, −1]. Zero pad the DFT of this vector to a length of 100, and call the result S. Scale S according to (d), take the inverse DFT, and plot the result. Does the new time-domain signal $s[n]$ look like a square wave? Explain.