

## Answer Key to HW 11

1) a) 
$$X[k] = \sum_{n=0}^3 x[n]e^{-j(\frac{\pi}{2})kn} = 1 - 2e^{-j(\frac{\pi}{2})k} + 2e^{-j(\frac{\pi}{2})2k} - e^{-j(\frac{\pi}{2})3k}$$

$$X[0] = 0, \quad X[1] = -1 + j, \quad X[2] = 6, \quad X[3] = -1 - j.$$

b) 
$$\Omega_0 = \frac{2\pi}{N} \leq 0.1 \text{ or } N \geq 62.8319.$$

$$X[k] = \sum_{n=0}^3 x[n]e^{-j(\frac{2\pi}{63})kn} = 1 - 2e^{-j(\frac{2\pi}{63})k} + 2e^{-j(\frac{2\pi}{63})2k} - e^{-j(\frac{2\pi}{63})3k}$$

2) a)  $X[5] = X^*[3] = 0, \quad X[6] = X^*[2] = -2j, \quad \text{and} \quad X[7] = X^*[1] = 0$

b) 
$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]e^{j\Omega_0kn} = \frac{1}{8} \left[ 1 + 2je^{j\pi n/2} + e^{j\pi n} - 2je^{j3\pi n/2} \right]$$

$$x[n] = [0.25 \quad -0.5 \quad 0.25 \quad 0.5 \quad 0.25 \quad -0.5 \quad 0.25 \quad 0.5]$$

3) a)  $X[k] = [4 \quad 0 \quad 0 \quad 0]$

b) 
$$\begin{aligned} X(\Omega) &= \frac{1}{N} \sum_{k=0}^{N-1} X[k] \frac{\sin\left(\frac{\Omega N - 2\pi k}{2}\right)}{\sin\left(\frac{\Omega N - 2\pi k}{2N}\right)} e^{-j(\Omega N - 2\pi k)(N-1)/2N} \\ &= \frac{1}{4} \sum_{k=0}^3 4\delta[k] \frac{\sin\left(\frac{\Omega 4 - 2\pi k}{2}\right)}{\sin\left(\frac{\Omega 4 - 2\pi k}{8}\right)} e^{-j(\Omega 4 - 2\pi k)(3)/8} \\ &= \frac{\sin(2\Omega)}{\sin\left(\frac{\Omega}{2}\right)} e^{-j3\Omega/2}. \end{aligned}$$

c) 
$$X[k] = \begin{bmatrix} 4 & 1.81 - 2.49i & -0.31 - 0.95i & 0.69 + 0.23i & 0.81 - 0.59i & 0 \\ & 0.81 + 0.59i & 0.69 - 0.23i & -0.31 + 0.95i & 1.81 + 2.49i & \end{bmatrix}$$

$$x[n] = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$