Answer Keys to HW 1: Chapter 3

1)

(a) $f_a = f_0 = 8$ Hz.

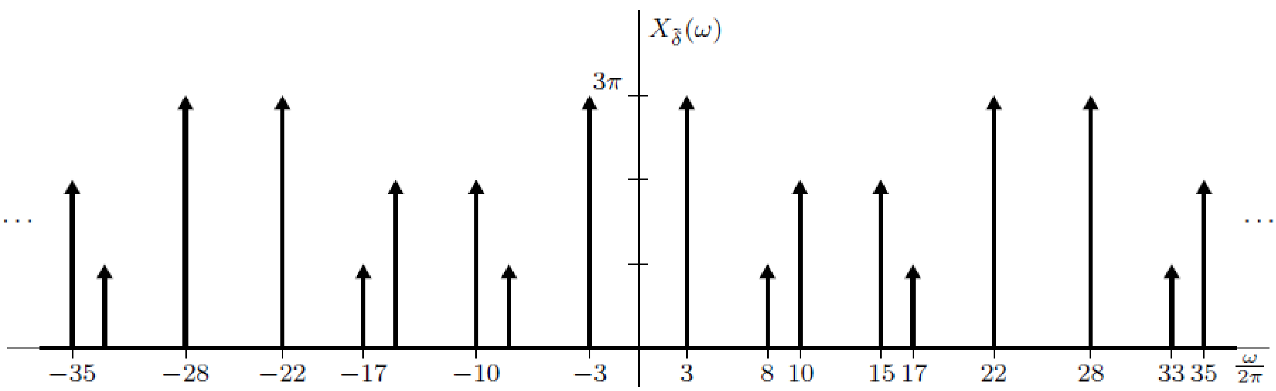
(b) Since f_a is negative, we can also report apparent frequency as $|f_a| = 8$ Hz.

(c) $f_a = \langle f_0 + F_s/2 \rangle_{F_s} - F_s/2 = 1$ Hz.

(d) Since f_a is negative, we can also report apparent frequency as $|f_a| = 8$ Hz.

2)

$$X(\omega) = \pi\left[3\delta(\omega \pm 6\pi)) + \delta(\omega \pm 16\pi)) + 2\delta(\omega \pm 20\pi))\right].$$



A sampling rate 25% below the Nyquist rate is $F_s = 15$ Hz.
Thus, the output contains 3, 5, and 7 Hz frequency components when $F_s = 15$ Hz.

3)

$$x(t) = \sin(0.7\pi t)\cos(0.5\pi t) = \frac{1}{2}\sin(2\pi 0.1t) + \frac{1}{2}\sin(2\pi 0.6t).$$

Using a sampling frequency of $F_s = 1$ Hz, the apparent frequencies of $\frac{1}{2}\sin(2\pi 0.1t)$ and $\frac{1}{2}\sin(2\pi 0.6t)$ are $f_a = \langle 0.1 + 0.5\rangle_1 - 0.5 = 0.1$ Hz and $f_a = \langle 0.6 + 0.5\rangle_1 - 0.5 = -0.4$ Hz, respectively.

When the sampled signal is passed through an ideal high-pass digital filter with cutoff frequency of $\frac{3}{10}F_s = 0.3$ Hz, the 0.1 Hz component is eliminated and the -0.4 Hz component is retained.

If the highpass digital filter output is reconstructed at a rate $F_s = 1$ Hz, the resulting signal is clearly $-\frac{1}{2}\sin(2\pi 0.4t)$. However, when the data of the highpass digital filter is reconstructed at a rate $F_s = \frac{1}{2}$ Hz (twice as slow as a 1 Hz rate), the signal is time expanded (frequency compressed) to

$$y(t) = -\frac{1}{2}\sin(2\pi 0.2t).$$

4)

(a) $F_s = 2(40) = 80$ kHz (samples/s).

(b) $F_s = 2(30) = 60$ kHz (samples/s).

(c) $F_s = 2(2(120)) = 480$ kHz (samples/s).

(d) $F_s = 2(40 + 60) = 200$ kHz (samples/s).

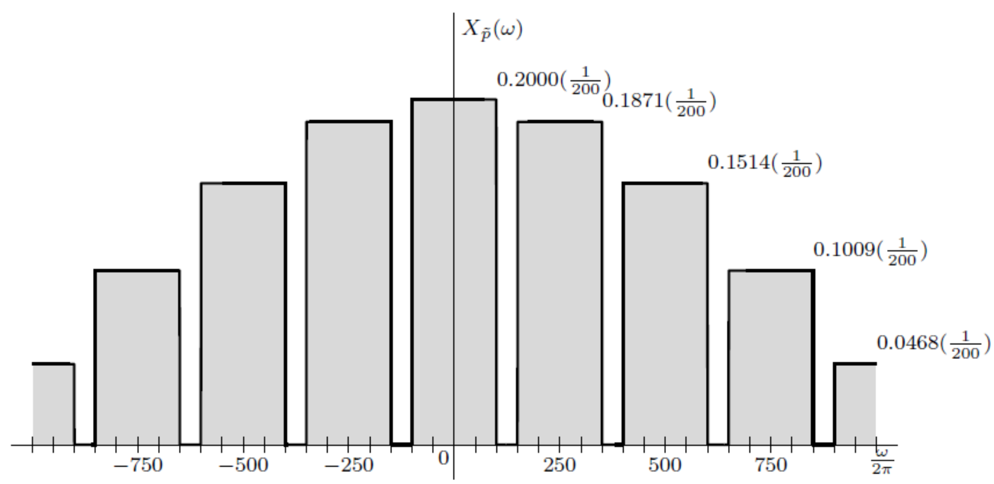
(e) $F_s = 2(40) = 80$ kHz (samples/s).

5)

$$x(t) = \mathrm{sinc}(200t) \Longleftrightarrow \frac{1}{200}\Pi\left(\frac{\omega}{400\pi}\right) = X(\omega).$$

$$X_{\tilde{p}}(\omega) = \sum_{k=-\infty}^{\infty} \tilde{P}_k X(\omega - k\omega_s).$$

$$X_{\tilde{p}}(\omega) = \sum_{k=-\infty}^{\infty} 0.2\mathrm{sinc}(0.2k)\frac{1}{200}\Pi\left(\frac{\omega - k\omega_s}{400\pi}\right).$$



y(t) = 0.2 x(t). If B> 150 Hz, the filter will pick up the unwanted spectral components from the k ≠ 0 replicates, and the output will be distorted.

6)

$$h(t) = u(t) - u(t-T) = \Pi\left(\frac{t - T/2}{T}\right). \qquad \hat{x}(t) = \sum_{n=-\infty}^{\infty} x(nT)h(t - nT) = \sum_{n=-\infty}^{\infty} x(nT)\Pi\left(\frac{t - nT - T/2}{T}\right)$$