Answer Keys to HW 2: Chapter 3

1)

(a) $(46666\frac{2}{3} < F_s < 50000)$, $(70000 < F_s < 100000)$, and $(140000 < F_s)$.

Clearly, the minimum permissible sampling rate is $F_{s,\min} = 46666\frac{2}{3}$ Hz.

(b) $(11250 < F_s < 11666\frac{2}{3})$, $(15000 < F_s < 17500)$, $(22500 < F_s < 35000)$, and $(45000 < F_s)$.

Clearly, the minimum permissible sampling rate is $F_{s,\min} = 11250$ Hz.

(c) $(25000 < F_s < 30000)$ and $(50000 < F_s)$.

Clearly, the minimum permissible sampling rate is $F_{s,\min} = 25000$ Hz.

(d) $20000 < F_s$.
Clearly, the minimum permissible sampling rate is $F_{s,\min} = 20000$ Hz.

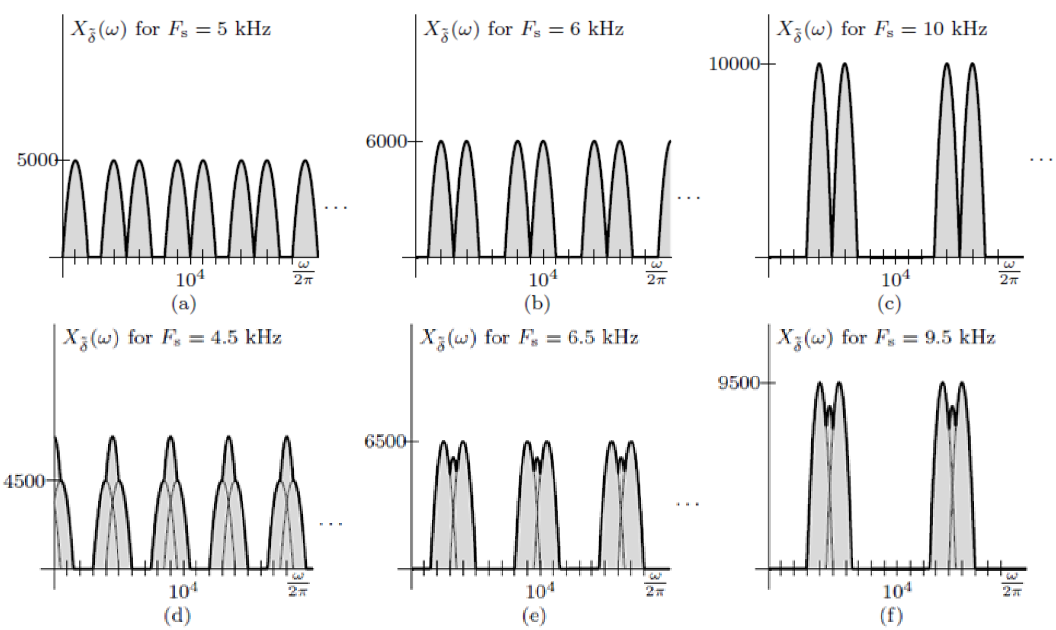
2)

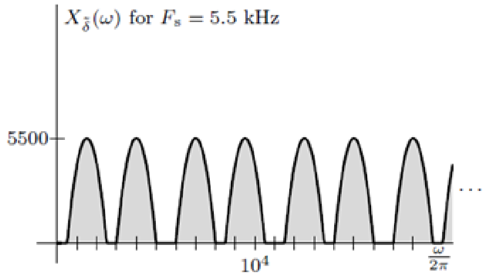
(a) The permissible sampling rates are $(5000 < F_s < 6000)$ and $(10000 < F_s)$.

(b) Sampling frequencies $F_s = 5$, 6, and 10 kHz are each (barely) within the permissible ranges computed in part (a). As shown in Figs. S3.3-4a, S3.3-4b, and S3.3-4c, respectively, the corresponding sampled signal spectra $X_{\tilde{\delta}}(\omega)$ have touching, but not overlapping, replicates. Thus, the original signal $x(t)$ can be reconstructed by passing the sampled signals through ideal bandpass filters with passband between 3 and 5 kHz and gains $T = \frac{1}{5000}, \frac{1}{6000}$, and $\frac{1}{10000}$, respectively.

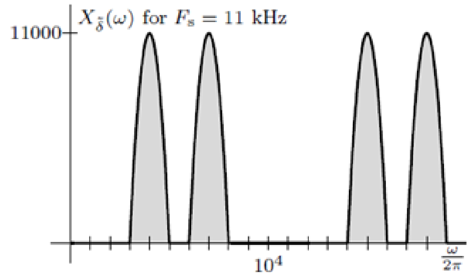
(c) Figures S3.3-4d, S3.3-4e, and S3.3-4f show the sampled signal spectra $X_{\tilde{\delta}}(\omega)$ when the sampling rates are adjusted from 5 kHz to 4.5 kHz, from 6 kHz to 6.5 kHz, and from 10 kHz to 9.5 kHz. In each case, the adjusted sampling rates no longer fall within the permissible ranges computed in part (a). Replicates overlap and distort the underlying signal spectrum. Thus, it is no longer possible to recover the original signal $x(t)$ through simple bandpass filtering.

(d) Figures S3.3-4g and S3.3-4h show the sampled signal spectra $X_{\tilde{\delta}}(\omega)$ when the sampling rates are 5.5 kHz and 11 kHz, respectively. Both rates fall neatly within the permissible ranges computed in part (a). Although the guard bands for the $F_s = 11$-kHz case are larger (meaning it is easier to design and implement the reconstruction filter), the $F_s = 5.5$-kHz sampling rate results in reasonable guard bands and is half the rate as the $F_s = 11$-kHz case. This substantial reduction in operating rate combined with reasonable guard bands makes $F_s = 5.5$ kHz the preferable rate.





(g)



(h)

3)

(a) The Nyquist rate for a 4.5 MHz signal is $2 \times 4.5 \times 10^6 = 9$ MHz. Exceeding the Nyquist rate by 20% yields a sampling rate of

$$F_s = 1.2 \times 9 = 10.8 \text{ MHz}.$$

(b) Since the desired number of levels is $1024 = 2^{10}$, 10 binary digits (bits) are needed to encode each sample.

(c) To transmit the TV signal requires a bit rate of $10.8 \times 10^6 \times 10 = 108 \times 10^6$ or 108 Mbits/s.

4)

(a) a $(B = 10)$-bit ADC is needed to meet error requirements.

(b) Nyquist requires that sampling be at least twice the highest frequency, so the maximum bandwidth of the analog input is $f_{\max} = \frac{5660}{2} = 2830$ Hz.

(c) As designed in the parts (a) and (b), this system cannot operate properly with input audio that has a bandwidth of 3500 Hz. This is because the 3500 Hz bandwidth of the streamed audio exceeds the maximum (alias-free) bandwidth of the system (2830 Hz), and the sample rate of 5660 samples/s will result in aliasing errors and signal distortion.

5)

$$\text{SQNR [dB]} = 10 \log_{10} \left(\tfrac{3}{2} 4^B\right) \geq 47.$$

$B \geq 7.514 \qquad B = 8$

for B = 8 the $\text{SQNR [dB]} = 48.165$ dB

6)

a) 0.3125   b) 3.75   c) 0110   d) -1.875