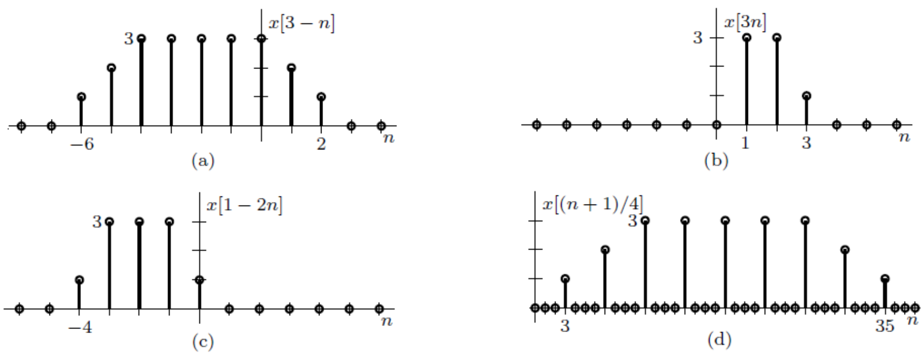
## Answer Keys to HW 3

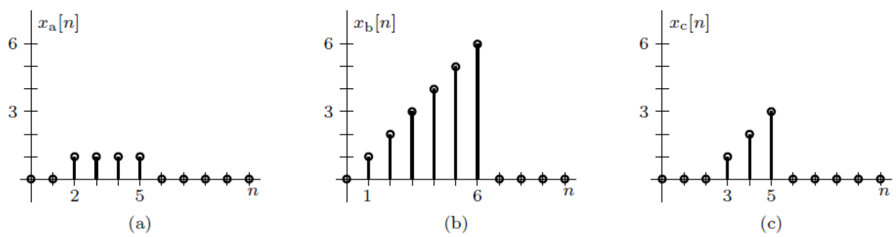
**1)**




2)

$$\theta = \Omega N \qquad \text{for any integer } N.$$

3)




4)

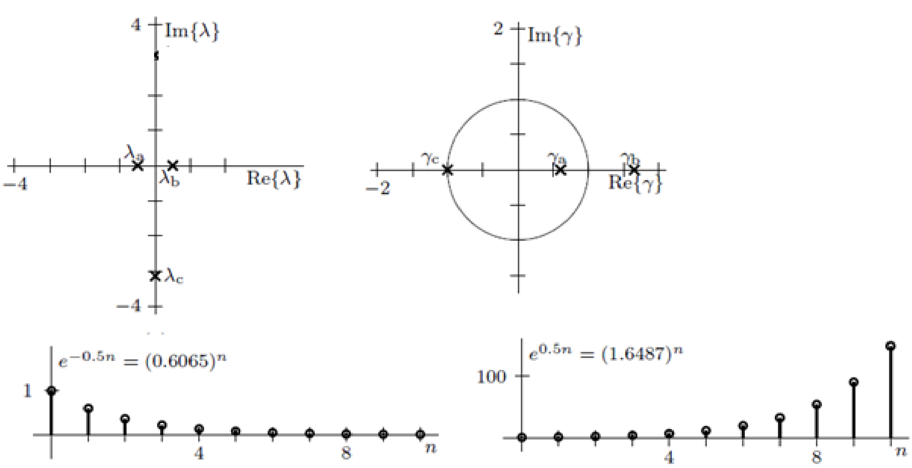
$$x_a[n] = (n+3)\,(u[n+3] - u[n]) + (3-n)\,(u[n] - u[n-4])\,.$$

$$x_a[n] = (n+3)\,(u[n+2] - u[n]) + (3-n)\,(u[n] - u[n-3])\,.$$

$$x_c[n] = 2n\,(u[n] - u[n-4]) + 2n\,(u[-n] - u[-n-4])\,.$$

$$x_c[n] = 2n\,(u[n+3] - u[n-4])\,.$$

5)



(a) Since $e^{-0.5n} = (0.6065)^n$, $\lambda_a = -0.5$ and $\gamma_a = 0.6065$,

(b) Since $e^{0.5n} = (1.6487)^n$, $\lambda_b = 0.5$ and $\gamma_b = 1.6487$,

(c) Since $e^{-j\pi n} = (-1)^n$, $\lambda_c = -j\pi$ and $\gamma_c = -1$,

(d) Since $e^{-(1+j\pi)n} = (-\frac{1}{e})^n = (-0.3679)^n$, $\lambda_a = -(1+j\pi)$ and $\gamma_a = -\frac{1}{e} = -0.3679$,

(e) Since $e^{(1-j\frac{\pi}{3})n} = (1.3591 - 2.3541j)^n$, $\lambda_f = 1 - j\frac{\pi}{3}$ and $\gamma_f = 1.3591 - 2.3541j$,

6)

$\omega_0 = 60\pi + 200\pi k.$

the three smallest values of $|\omega_0|$ are $60\pi$, $140\pi$, and $260\pi$ rad/s.

7)

$F_s = \dfrac{100}{2k+1}$. Using $k = [0, 1, 2]$,

the three largest values of $F_s$ are 100, $33\frac{1}{3}$, and 20 Hz.

8)

From the perspective of apparent frequency, exponentials are no different than sinusoids. Thus, we compute the apparent frequency of each complex exponential as $\Omega_a = \langle \Omega + \pi \rangle_{2\pi} - \pi$. This provides frequencies in the fundamental range, $-\pi \le \Omega_a < \pi$. Frequencies in the range $0 \le \Omega < 2\pi$ can be obtained by adding $2\pi$ to all values $\Omega_a < 0$.

(a) $e^{j(0.2\pi n+\theta)}$.

(b) $e^{0n} = 1$.

(c) $e^{-j1.95n}$. in the range $0 \le \Omega < 2\pi$, $e^{j4.3332n}$.

(d) $e^{-j0.7\pi n}$. in the range $0 \le \Omega < 2\pi$, $e^{j1.3\pi n}$.

9)

a)

$x(nT) = 10\cos(2\pi 1000nT) + 3.1623\cos(2\pi 1500nT - 1.1071)$.

$$x[n] = 10\cos\left(\frac{\pi}{2}n\right) + 3.1623\cos\left(\frac{3\pi}{4}n - 1.1071\right).$$

b) T < 1/5000