ENGR 4333/5333: Digital Signal Processing

HW 8: Ch 6

1) Using the DTFT method, find the zero-state response $y[n]$ of a causal system with frequency response

$$H(\Omega) = \frac{e^{j\Omega} - 0.5}{(e^{j\Omega} + 0.5)(e^{j\Omega} - 0.1)} \quad \text{and input} \quad x[n] = 3^{-(n+1)}u[n].$$

2) Find and sketch the magnitude and phase responses for an accumulator system described as

$y[n] - y[n-1] = x[n].$

Determine the responses of this system to the following input sinusoids:

(a) $x_a[n] = \cos(0.1n)$ **(b)** $x_b[n] = \sin(\pi n/2)$ **(c)** $x_c(t) = \cos(10^6 \pi t)$ sampled at rate $F_s = 1.25$ MHz

3) Determine and sketch the magnitude and phase responses for an LTID system specified by the equation

$y[n+1] - 0.5y[n] = x[n+1] + 0.8x[n]$. Find the system output $y[n]$ for the input $x[n] = \cos(0.5n - \pi/3)$.

4) An LTID system impulse response is $h[n] = (0.5)^n u[n]$. Find the system output $y[n]$ if the input is the bandpass signal $x[n] = v[n]\cos(3\pi n/4)$, where $v[n]$ is a narrowband signal of bandwidth 0.02π. Use the criterion that magnitude-response variations within $\pm 5\%$ and time-delay (group delay) variations within $\pm 3\%$ are considered distortionless.

5) A CT signal $x_c(t)$, bandlimited to 20 kHz, is sampled at 40 kHz to produce

$$x[n] = \begin{cases} 1 & n = -4, -2, 0, 2, 4 \\ -2 & n = -3, -1, 1, 3 \\ 0 & \text{otherwise} \end{cases}$$

Determine the CTFT $X_c(\omega)$.

6) (a) For the signal $x[n] = 3/16 \operatorname{sinc}(3n/16)$, find the $M = 2$ downsampled signal $x_{\downarrow}[n]$. What is the maximum factor M that still permits lossless (no aliasing) downsampling?

(b) Find the spectrum $X_{\downarrow}(\Omega)$ from $x_{\downarrow}[n]$ found in part (a). Verify the result by finding $X_{\downarrow}(\Omega)$ using Eq. (6.68).

(c) The downsampled signal $y[n] = x_{\downarrow}[n]$, found in part (a), is ideally interpolated by factor $L = 2$. Find the DTFT of the interpolated signal $y_i[n]$. Verify this result by finding $Y_i(\Omega)$ using Eq. (6.72). Is $y_i[n] = x[n]$? Why or why not?

7) (a) For the signal $x[n] = n\gamma^n u[n]$, find the $L = 3$ interpolated signal $x_i[n]$. Determine and sketch the spectrum $X_i(\Omega)$ of the interpolated signal. Assume an ideal interpolation filter.

(b) The interpolated signal $y[n] = x_i[n]$, found in part (a), is downsampled by factor $M = 3$. Determine and sketch the spectrum of the downsampled signal $y_{\downarrow}[n]$. Is $y_{\downarrow}[n] = x[n]$? Why or why not?