Answer Keys to HW 8

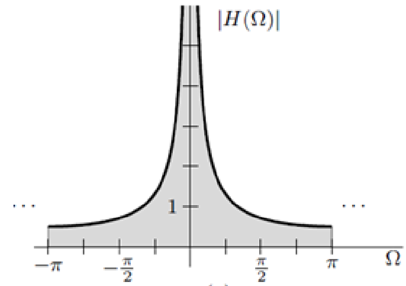
1)

$$Y(\Omega) = \frac{-\frac{2}{3}e^{j\Omega}}{e^{j\Omega}+\frac{1}{2}} - \frac{\frac{2}{7}e^{j\Omega}}{e^{j\Omega}-\frac{1}{3}} + \frac{\frac{20}{21}e^{j\Omega}}{e^{j\Omega}-\frac{1}{10}}$$

$$y[n] = \left(-\frac{2}{3}\left(-\frac{1}{2}\right)^n - \frac{2}{7}\left(\frac{1}{3}\right)^n + \frac{20}{21}\left(\frac{1}{10}\right)^n\right)u[n]$$

2)

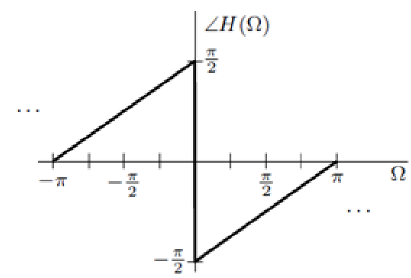
$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1-e^{-j\Omega}}$$



$$y_{\mathrm{a}}[n] = 10\ \cos(0.1n - 1.521)$$

$$y_{\mathrm{b}}[n] = 0.7071\sin\left(\frac{\pi n}{2} - 0.7854\right)$$

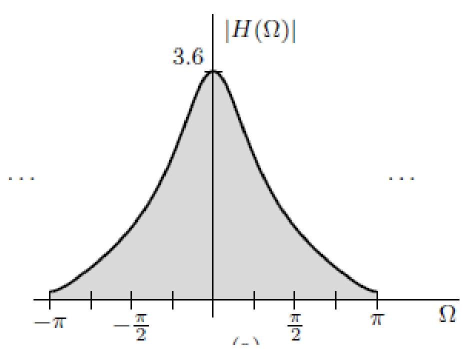
$$y_{\mathrm{c}}[n] = 10\ \cos\left(\frac{4\pi n}{5} - 0.3142\right)$$

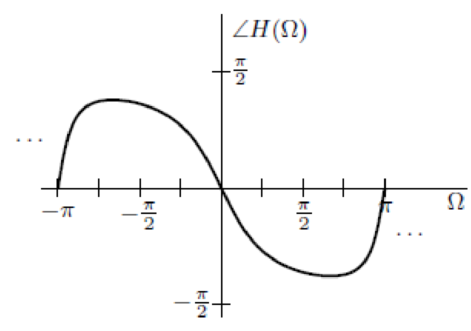


3)

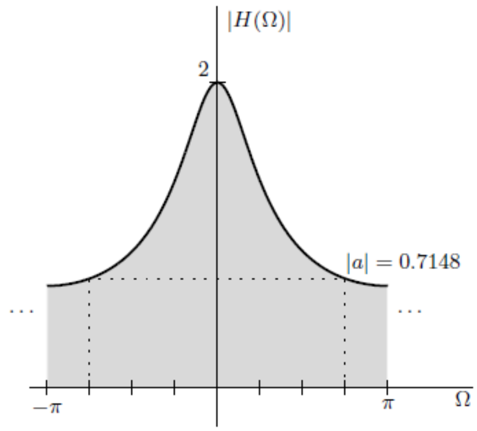
$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{e^{j\Omega}+\frac{4}{5}}{e^{j\Omega}-\frac{1}{2}}$$

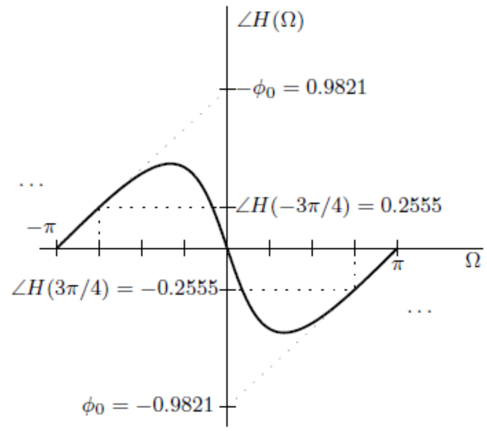
$$y[n] = 2.8590\cos(0.5n - \frac{\pi}{3} - 0.6253) = 2.8590\cos(0.5n - 1.6725)$$





4) $$H(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - \frac{1}{2}} = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}}$$





$$H_{\text{glp}}(\Omega) = \begin{cases} |a|e^{j(\phi_0 - \Omega n_g)} & \frac{3\pi}{4} - 0.02\pi \le \Omega \le \frac{3\pi}{4} + 0.02\pi \\ |a|e^{j(-\phi_0 - \Omega n_g)} & -\frac{3\pi}{4} - 0.02\pi \le \Omega \le -\frac{3\pi}{4} + 0.02\pi \end{cases}$$

$$\Omega_c = \frac{3\pi}{4},$$

$$|a| = |H(3\pi/4)| = 0.7148, \qquad n_g(\Omega) = -\frac{d}{d\Omega}\angle H(\Omega) = \frac{2\cos(\Omega) - 1}{5 - 4\cos(\Omega)}$$

$$n_g = \frac{2\cos(3\pi/4) - 1}{5 - 4\cos(3\pi/4)} = -0.3084,$$

$$\phi_0 = \angle H(3\pi/4) + \frac{3\pi}{4}n_g = -0.9821$$

$$\begin{aligned} y[n] &= |a|v[n - n_g]\cos\left[\Omega_c(n - n_g) + \phi_0\right] \\ &= 0.7148v[n + 0.3084]\cos\left[\frac{3\pi}{4}(n + 0.3084) - 0.9821\right] \end{aligned}$$

5)

$$\begin{aligned} X_c(\omega) &= \begin{cases} TX(\omega T) & |\omega| \le \pi/T \\ 0 & \text{otherwise} \end{cases} \\ &= \frac{1}{40000}\left[1 - 4\cos\left(\frac{\omega}{40000}\right) + 2\cos\left(\frac{2\omega}{40000}\right) - 4\cos\left(\frac{3\omega}{40000}\right) + 2\cos\left(\frac{4\omega}{40000}\right)\right]\Pi\left(\frac{\omega}{80000\pi}\right) \end{aligned}$$

6a) $$x_{\downarrow}[n] = x[2n] = \frac{3}{16}\text{sinc}(3n/8).$$

$M_{\text{max}} = 5$ is the maximum downsampling factor.

b) the DTFT of $x_{\downarrow}[n] = \frac{3}{16}\text{sinc}(3n/8)$ is $X_{\downarrow}(\Omega) = \sum_{k=-\infty}^{\infty} \frac{1}{2}\Pi\left(\frac{\Omega - 2\pi k}{2(3\pi/8)}\right)$

$$X_{\downarrow}(\Omega) = \frac{1}{2}\sum_{k=-\infty}^{\infty}\left[\Pi\left(\frac{\Omega - 4\pi k}{2(3\pi/8)}\right) + \Pi\left(\frac{\Omega - 2\pi - 4\pi k}{2(3\pi/8)}\right)\right]$$

c) $$Y_{\text{i}}(\Omega) = X(\Omega) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{\Omega - 2\pi k}{2(3\pi/16)}\right)$$

From Eq. (6.72), $Y_{\text{i}}(\Omega) = Y(M\Omega)H_{\text{i}}(\Omega)$,

where $$H_{\text{i}}(\Omega) = M\sum_{k=-\infty}^{\infty} \Pi\left(\frac{\Omega - 2\pi k}{2\pi/M}\right)$$

Thus, $$Y_{\text{i}}(\Omega) = \left[\sum_{k=-\infty}^{\infty} \frac{1}{2}\Pi\left(\frac{2\Omega - 2\pi k}{2(3\pi/8)}\right)\right]\left[2\sum_{k=-\infty}^{\infty} \Pi\left(\frac{\Omega - 2\pi k}{\pi}\right)\right]$$
$$= \left[\sum_{k=-\infty}^{\infty} \Pi\left(\frac{\Omega - \pi k}{3\pi/8}\right)\right]\left[\sum_{k=-\infty}^{\infty} \Pi\left(\frac{\Omega - 2\pi k}{\pi}\right)\right]$$

7) a) $$X_{\text{i}}(\Omega) = X(3\Omega)H_{\text{i}}(\Omega) = \frac{\gamma e^{j3\Omega}}{(e^{j3\Omega} - \gamma)^2}\, 3\,\Pi\left(\frac{\Omega}{2\pi/3}\right)$$

$$x_{\text{i}}[n] = \sum_{k=0}^{\infty} k\gamma^k \text{sinc}\left(\frac{n}{3} - k\right)$$

b) The cascade of an ideal interpolator followed by a downsampler, both of factor L, is an identity system. Thus,

$$y_{\downarrow}[n] = x[n] = n\gamma^n u[n] \qquad \text{and} \qquad Y_{\downarrow}(\Omega) = X(\Omega) = \frac{\gamma e^{j\Omega}}{(\gamma - e^{j\Omega})^2}$$