

Answer Keys to HW 9

1) a)  $X_a(z) = \frac{-1.2z}{z^2 - 2.8z + 1.6}, \quad 0.8 < |z| < 2.$

b)  $X_b(z) = \frac{2z^2 - 5z}{z^2 - 5z + 6}, \quad 2 < |z| < 3.$

c)  $X_c(z) = \frac{-1.5z}{z^2 - 2.5z + 1}, \quad 0.5 < |z| < 2$

2) a)  $X_a(z) = \frac{z}{z^m(z-1)}, \quad |z| > 1$

b)  $X_b(z) = \frac{\gamma z}{(z-\gamma)^2}, \quad |z| > |\gamma|$

3) a)  $x_a[n] = \frac{2}{5}(-2)^n u[n] + \frac{3}{5}(0.5)^n u[n] \quad \text{when } |z| > 2.$

$$x_a[n] = -\frac{2}{5}(-2)^n u[-n-1] + \frac{3}{5}(0.5)^n u[n] \quad \text{when } 0.5 < |z| < 2.$$

$$x_a[n] = -\frac{2}{5}(-2)^n u[-n-1] - \frac{3}{5}(0.5)^n u[-n-1] \quad \text{when } |z| < 0.5.$$

b)  $x_b[n] = 2 \cos\left(\frac{\pi}{2}n\right) u[n] - \delta[n] \quad \text{when } |z| > 1.$

$$x_b[n] = -2 \cos\left(\frac{\pi}{2}n\right) u[-n-1] - \delta[n] \quad \text{when } |z| < 1$$

4) a)  $x_a[n] = [2(2)^n - (3)^n] u[n].$

b)  $x_b[n] = \left[ (2)^n - \frac{1}{3}(3)^n \right] u[n] - \frac{2}{3}\delta[n]$

c)  $x_c[n] = [3(-1)^n - 3(2)^n + 2n(2)^n] u[n].$

d)  $x_d[n] = 2 \cos\left(\frac{\pi}{3}n + \frac{\pi}{3}\right) u[n].$

5)  $X(z) = \frac{1}{z^7(z-1)^2} [z^8 - 2z^4 + 1]$

or

$$X(z) = \frac{z^6 + 2z^5 + 3z^4 + 4z^3 + 3z^2 + 2z + 1}{z^7}.$$

6) a)  $y[n] = \left[ \frac{1}{3} + \frac{5}{2} (0.5)^n - \frac{4}{3} (0.25)^n \right] u[n]$

b)  $y_{\text{zir}}[n] = \left( -1 + \frac{1}{2} (0.5)^n \right) u[n].$

$y_{\text{zsr}}[n] = \left[ \frac{4}{3} + 2 (0.5)^n - \frac{4}{3} (0.25)^n \right] u[n].$

c)  $y_{\text{transient}}[n] = \left[ \frac{5}{2} (0.5)^n - \frac{4}{3} (0.25)^n \right] u[n]$

$y_{\text{steady-state}}[n] = \frac{1}{3} u[n].$

7) a) Since the system poles are located at  $z = 0.8$  and  $z = -0.2$ , the system is asymptotically stable.

b)  $y_{\text{zsr}}[n] = [1.3199(e)^n - 1.1336(0.8)^n - 0.1863(-0.2)^n] u[n].$

c)  $y[n] - 0.6y[n - 1] - 0.16y[n - 2] = x[n - 1].$

d)  $h[n] = [(0.8)^n - (-0.2)^n] u[n].$

