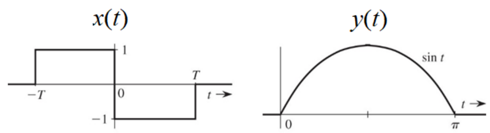
ENGR 3323: Signals and Systems

HW 11_Ch7

Q1) Find the Fourier transforms (FT) of the signals shown below using only the time-shifting property and the FT Table.



$X(\omega) = \frac{j4}{\omega} \sin^2\left(\frac{\omega T}{2}\right)$ $\qquad Y(\omega) = \frac{1}{1-\omega^2}\left(1+e^{-j\pi\omega}\right)$

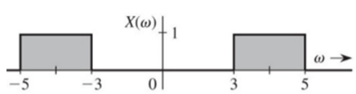
Q2) The signals below are modulated signals with carrier cos(10t). Find the Fourier transforms of these signals by using the Fourier transform properties and the FT table. Sketch the amplitude and phase spectra for the signals.

$X(\omega)$

$Y(\omega) = X(\omega)\, e^{-j2\pi\omega}$

$X(\omega) = \frac{\pi}{2}\left\{ \operatorname{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \operatorname{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$

Q3) Use the frequency-shifting property and the FT Table to find the inverse Fourier transform of the spectra shown below



$x(t) = \frac{2}{\pi} \operatorname{sinc}(t) \cos 4t$

Q4) For a stable LTIC system with transfer function $H(s) = 1/(s+1)$ find the (zero-state response) if the input $x(t)$ is

a) $e^{-2t}u(t)$ b) $e^{-t}u(t)$ c) $e^{t}u(-t)$ d) $u(t)$

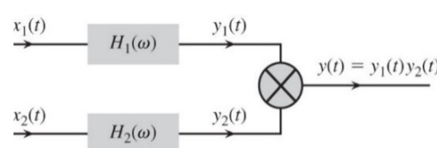
$x_a(t) = [e^{-t} - e^{-2t}]u(t)$ $\qquad x_b(t) = t e^{-at} u(t)$

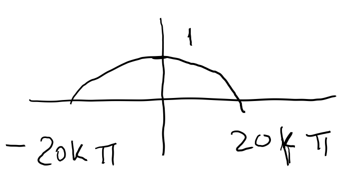
$x_c(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{t}u(-t)$ $\qquad x_d(t) = (1-e^{-t})u(t)$

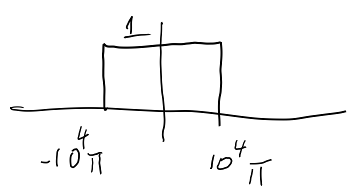
Q5) Signals $x_1(t) = 10^4\text{rect}(10^4 t)$ and $x_2(t) = \delta(t)$ are applied at the inputs of the ideal low-pass filters shown below $H_1(\omega) = \text{rect}(\omega/40{,}000\pi)$ and $H_2(\omega) = \text{rect}(\omega/20{,}000\pi)$. The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$.

a) Sketch $X_1(\omega)$ and $X_2(\omega)$. b) Sketch $H_1(\omega)$ and $H_2(\omega)$.

c) Sketch $Y_1(\omega)$ and $Y_2(\omega)$. d) Find the bandwidths of $y_1(t)$, $y_2(t)$, and $y(t)$.







$BW_{y_1} = 10\text{k}$

$BW_{y_2} = 5\text{k}$

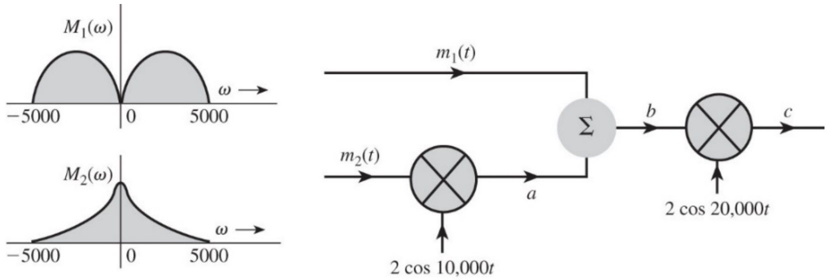
$BW_y = 15\text{k}$

Q6) For the signal $x(t) = \frac{2a}{t^2+a^2}$ determine the essential bandwidth B (in hertz) of $x(t)$ such that the energy contained in the spectral components of $x(t)$ of frequencies below B Hz is 99% of the signal energy E_x.

$$B = \frac{0.366}{a} \text{ Hz}$$

Q7) Figure below shows a scheme to transmit two signals $m_1(t)$ and $m_2(t)$ simultaneously on the same channel (without causing spectral interference). Such a scheme, which transmits more than one signal, is known as signal *multiplexing.* In this case, we transmit multiple signals by sharing an available spectral band on the channel, and, hence, this is an example of the *frequency-division* multiplexing. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency 20,000 rad/s. The modulated signal at point c is now transmitted over the channel.

a) Sketch the spectra at points *a, b,* and *c*. b) What must be the minimum bandwidth of the channel?
c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point *c*.



(a) (b)

at node c

at Node b