ENGR 3323: Signals and Systems

HW 8_Ch6 Answer Keys

Q1) For the periodic signals $x(t)$ and $y(t)$ shown below:

a) Here, $T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$. Because of even symmetry, all the sine terms are zero.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right) + b_n \sin\left(\frac{n}{5}t\right)$$

$$a_0 = \frac{1}{5} \quad \text{(by inspection)}$$

$$a_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos\left(\frac{n}{5}t\right)\, dt = \frac{1}{5\pi}\left(\frac{5}{n}\right)\sin\left(\frac{n}{5}t\right)\Big|_{-\pi}^{\pi} = \frac{2}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

$$b_n = \frac{2}{10\pi}\int_{-\pi}^{\pi} \sin\left(\frac{n}{5}t\right)\, dt = 0 \quad \text{(integrand is an odd function of } t\text{)}$$

Since $b_n = 0$, $C_n = |a_n|$ and $\theta_n = \angle a_n$, both shown in Fig. S6.1-1b. The corresponding frequency ω is easily computed as $\omega_0 n = \frac{n}{5}$.

In this case, $T_0 = 2\pi$, $\omega_0 = 1$. Thus,

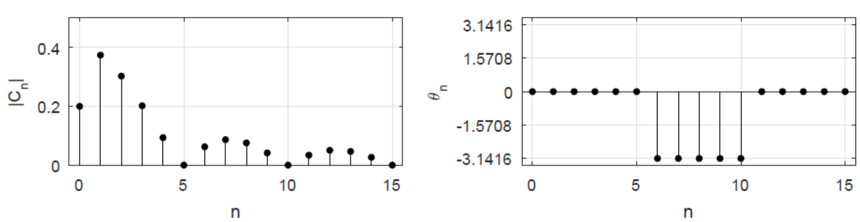
$$y(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \quad \text{with} \quad a_0 = 0.5 \quad \text{(by inspection)},$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\cos nt\, dt = 0, \qquad b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\sin nt\, dt = -\frac{1}{\pi n}$$

and

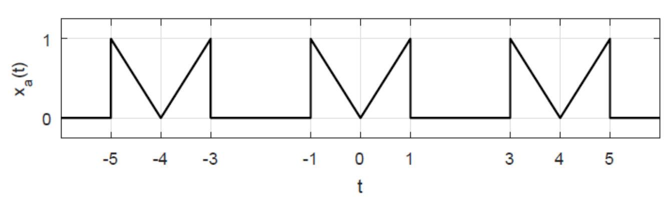
$$y(t) = 0.5 - \frac{1}{\pi}\left(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \frac{1}{4}\sin 4t + \cdots\right)$$
$$= 0.5 + \frac{1}{\pi}\left[\cos\left(t + \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2t + \frac{\pi}{2}\right) + \frac{1}{3}\cos\left(3t + \frac{\pi}{2}\right) + \cdots\right]$$

b)

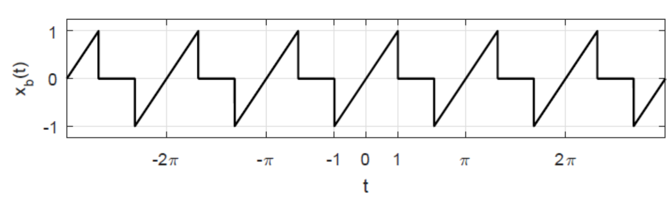


Q2)

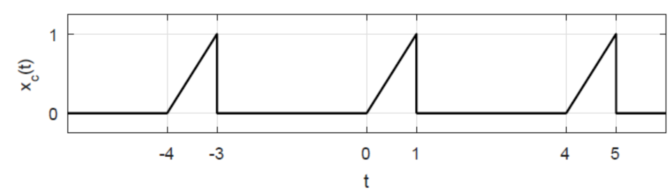
a)



b)



c)



Q3)

In each case the signal is periodic, it is because $x(t) = x(t+T)$ for all t. If $x(t) \neq x(t+T)$ for all t, then the signal is not periodic.

	(a)	(b)	(c)	(d)	(e)	(f)
Periodic?	Yes	Yes	No	Yes	No	Yes
ω_0	1	1		π		1
Period T	2π	2π		2		2π