ENGR 3323: Signals and Systems

HW 9_Ch6 Answer Keys

Q1)

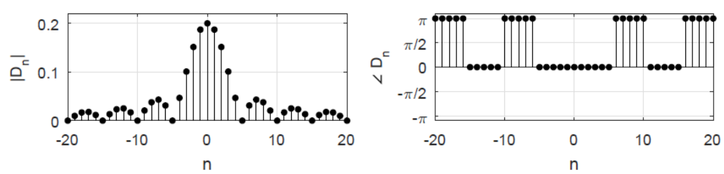
a) for $x(t)$

$$D_n = \frac{1}{5}\text{sinc}\left(\frac{n\pi}{5}\right)$$

for $y(t)$

$$D_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi} e^{-jnt}\, dt = \frac{j}{2\pi n},$$

b)

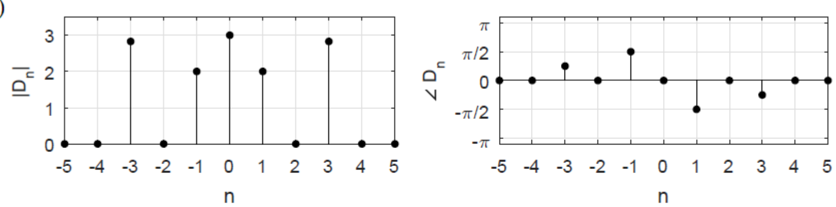


c) For x(t), the 5th hormonic will be less than 10% of the DC and the approximate power for the DC and the first five harmonics is P = 0.1805.

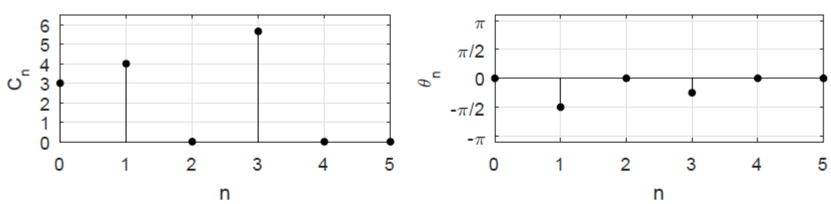
For y(t), the 2nd hormonic will be less than 10% of the DC and the approximate power for the DC and the first two harmonics is P = 0.3013.

Q2)

a)



b) $x(t) = 3 + 4\cos\left(t - \frac{\pi}{2}\right) + 4\sqrt{2}\cos\left(3t - \frac{\pi}{4}\right)$



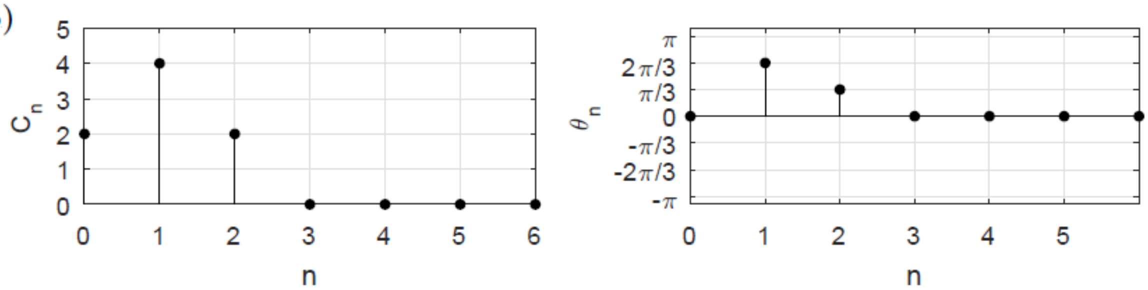
c) Since, the trigonometric series in part (b) is obtained from the exponential series in part (a), the two series are equivalent.

d) The lowest frequency in the spectrum is 0 and the highest frequency is 3 rad/s. Therefore, the bandwidth is 3 rad/s or $\frac{3}{2\pi}$ Hz.

Q3)

(a) $x(t) = 2 + 2e^{j(t+\frac{2\pi}{3})} + 2e^{-j(t+\frac{2\pi}{3})} + e^{j(2t+\frac{\pi}{3})} + e^{-j(2t+\frac{\pi}{3})}$

(b)



(c) $x(t) = 2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right)$

(d)
$$\begin{aligned} x(t) &= 2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right) \\ &= 2 + 2e^{j(t+\frac{2\pi}{3})} + 2e^{-j(t+\frac{2\pi}{3})} + e^{j(2t+\frac{\pi}{3})} + e^{-j(2t+\frac{\pi}{3})} \end{aligned}$$

Q4) For $x(t) = (2 + j2)e^{-j3t} + j2e^{-jt} + 3 - j2e^{jt} + (2 - j2)e^{j3t}$

$D_{-3} = 2 + j2$; $D_{-2} = 0$; $D_{-1} = j$; $D_0 = 3$; $D_1 = -j$; $D_2 = 0$; $D_3 = 2 - j2$

$$H(j\omega) = \frac{j\omega}{(-\omega^2 + 3) + j2\omega}$$

$$y(t) = \sum_{n=-\infty}^{\infty} D_n H(jn\omega_0) e^{jn\omega_0 t}$$

Q5)

(a) $$x(t) = \sum_{n=-\infty}^{\infty} \frac{1 - e^{-1}}{1 + j2\pi n} e^{j2\pi n t}$$

(b) $$H(j\omega) = \frac{j\omega}{j\omega + 1}$$

$$\begin{aligned} y(t) &= \sum_{n=-\infty}^{\infty} D_n H(j2\pi n) e^{j2\pi n t} \\ &= \sum_{n=-\infty}^{\infty} \frac{j2\pi n(1 - e^{-1})}{(1 + j2\pi n)^2} e^{j2\pi n t} \end{aligned}$$